

# Study of Memory effects in the reaction $^{136}\text{Xe} + ^{209}\text{Bi}$ at $E_{\text{lab}}$ 1130 MeV and 1420 MeV

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**Abstract:** The dissipative heavy-ion reactions have been studied extensively both from the theoretical as well as experimental points of view covering a wide range of energies [1]. The main objective of this communication is to point out and emphasis the influence of memory effects on the observable quantities like angular distributions  $d\sigma/d\theta_{\text{cm}}$ , energy distributions  $d\sigma/d\Delta E$  and element distributions  $d\sigma/dZ$  in the dissipative heavy-ion collisions. Dissipative collisions between two heavy nuclei are described in terms of a macroscopic dynamical model within the framework of a multi-dimensional Fokker- Planck Equation (FPE). The effect of two-body collisions leading to intrinsic equilibration has been treated phenomenologically employing the basic concept of dissipative diabatic dynamics (DDD). The reaction  $^{136}\text{Xe}$  (8.31 MeV/u) and (10.4 MeV/u) +  $^{209}\text{Bi}$  has been used as a prototype to study and demonstrate the memory effects for dissipation and diffusion processes. The results of these calculations comprise the mean values of deflection function  $\theta_{\text{cm}}$ , kinetic energy loss  $\Delta E$ , fragment charge  $Z$  and the differential cross sections  $d\sigma/d\theta_{\text{cm}}$ ,  $d\sigma/d\Delta E$ ,  $d\sigma/dZ$  and their comparison with the experimental data. Our calculated results for the mean values of deflection function, kinetic energy loss, angular distributions, energy distributions and element distributions illustrate a remarkable dependence on the memory effects and are consistent with the experimental data. It is found that the method of moment expansion, the only practicable approach available for solving the multi-dimensional Fokker - Planck Equation (FPE), diverges for creeping trajectories. I suggest an approximate treatment of such a creeping-motion in terms of appropriately reduced distribution function for the collective variables.

**Keywords:** heavy-ion reactions, DDD, FPE, time dependent Hartree-Fock (TDHF), differential cross sections & energy loss.

## 1 Introduction:

Heavy-ion reactions with energies of a few MeV per nucleon above the Coulomb barrier are well known [2, 3] to exhibit dissipative processes in nuclei. Such dissipative reactions are characterized by the dissipation of a large amount of kinetic energy and angular momentum of the relative motion into intrinsic excitations, as well as by the diffusion of nucleons between the two colliding heavy-ions, and cover the range between direct reactions and compound nuclear formation. Only a few degrees of freedom are involved in a direct reaction, whereas all degrees of freedom participate in compound nucleus formation [3, 4]. Thus dissipative reactions carry information regarding the relaxation processes leading simple nuclear states to more complex configurations. The collision process can be roughly divided into two stages [5, 6]: mutual approach of the nuclei, intrinsic(local) equilibration and slow relaxation of macroscopic degrees of freedom. The assumption of intrinsic equilibrium as produced by the residual two-body interactions, provides the common starting point of Markovian transport theories which have been used for the description of heavy-ion

dissipative collisions [7] and [8]. For a complete description of the nucleus-nucleus collision the TDHF calculations have to be complemented by the inclusion of two-body collisions [6, 9]. However, the inclusion of two-body collisions in the TDHF description is marred by the numerical complexities and extensive calculations are practically difficult to perform [10, 11]. As an alternative, it has been suggested earlier to include within a transport theoretical approach [5, 10] the main memory effects, so called the non-Markovian effects, which are due to long mean free path of the nucleons. The underlying theory is referred to as dissipative diabatic dynamics (DDD) and provides a natural extension of the current Markovian transport theories [12]. The DDD is characterized by a two step process. It ascribes [5, 10, 13] elastoplastic properties to nuclear matter and supplies a link between the description of giant vibrations (initial stage) and the overdamped motion (final stage of the reaction). Apart from the present approach, memory effects have been considered by several authors within the framework of linear response theory [14, 15, 16], and the results obtained for the friction coefficients do indicate that the memory effects may not be negligible. With the above in view; a detailed investigation of the reaction

$^{136}\text{Xe}$  (8.31 MeV/u) and (10.4 MeV/u) +  $^{209}\text{Bi}$  has been carried out within a phenomenological approach to the basic elements of DDD [11,18]. Our model uses a multi dimensional FPE, which is modified as compared to the standard treatments to include the memory effects due to intrinsic equilibration process. The paper is devoted to an extensive study of memory effects on the observables such as angular distribution  $d\sigma/d\theta_{\text{cm}}$ , energy distribution  $d\sigma/d\Delta E$  and element distribution  $d\sigma/dZ$ . The FPE describes the evolution of the probability distribution function defined in terms of the dynamical collective coordinates  $q_i$  and conjugate momenta  $p_i$ . Since the complete microscopic derivation of such a FPE is not yet available and the numerical solution of the FPE for multi dimensional probability distribution function is not practicable therefore I am bound to a phenomenological treatment of the equilibration effects.

## 2 Calculation and results for Xe+Bi system:

The results of reaction  $^{136}\text{Xe}$  (8.31 MeV/u) and (10.4 MeV/u) +  $^{209}\text{Bi}$  are presented in this communication with a special emphasis on mean values and differential cross-sections. In Figs. 1-6, I have shown the mean values for the deflection angle  $\theta_{\text{cm}}$ , kinetic energy loss  $\Delta E$  and Projectile charge  $Z$ , along with the differential cross sections for the three different cases (i) ELPL1, (ii) ELPL2 and (iii) DISLIM. I recall that the third case, DISLIM, describes the dissipative limit of the collision process and no consideration is made for the diabatic potential. The ELPL1 case describes the elastoplastic processes and the drift and diffusion coefficients, except those related to angular motions, are modified due to diabatic potential through the factor  $(1 - \chi(t))$ . The ELPL2 case also describes the elastoplastic processes whereby now all the drift and diffusion coefficients including those for angular motions are modified due to the diabatic potential through the factor  $(1 - \chi(t))$ . In above Figs. the three cases ELPL1, ELPL2 and DISLIM have been distinguished by solid, dashed and dotted lines respectively. In Figs. 2, 4 and 6 I have also displayed the experimental differential cross sections for the angular, energy and element distribution taken from ref. [4]. As the present study pertains to a description of dissipative processes, I have not included the parts of differential cross sections describing the quasi-elastic regime of energy loss. However, in Figs. 1, 3

and 5 the results for the mean values and associated variances (not the differential cross sections) have been displayed for even larger impact parameters to show the trend of impact parameters dependence.

First, I consider the impact parameter dependence of the deflection angle. The mean deflection angle  $\theta_{\text{cm}}$  is uniquely determined by the incident center of mass energy  $E_{\text{cm}}$  and impact parameter  $b$ . The mean values for deflection angle is defined through  $\theta_{\text{cm}} = \Pi - \langle \theta \rangle$ . For the lighter systems the nuclei are constrained by the attractive nuclear forces to the smaller scattering angles and therefore the trajectories are nuclear like. For very heavy systems  $^{136}\text{Xe} + ^{209}\text{Bi}$  the repulsion dominates and therefore the trajectories are Coulomb like. The mean deflection angle is positive for large impact parameters due to Coulomb scattering and tends to  $-\infty$  for very small impact parameters indicating the capture and fusion of two nuclei. There is no indication for negative deflection angle for the Xe + Bi system at  $E_{\text{lab}} = 1130$  MeV that has been observed for somewhat lighter system or higher bombarding energies above the Coulomb energy as indicated at  $E_{\text{lab}} = 1420$  MeV. When the bombarding energy is 1420 MeV for ELPL1 and ELPL2 cases the large negative deflection angles are observed for smaller impact parameters the interaction time is relatively large shows that the deflection angle becomes negative for the completely damped components. Calculated mean deflection angle  $\theta_{\text{cm}}$  as a function of the impact parameter  $b$  for three sets of calculations designated as elastoplastic cases (1) ELPL1 and (2) ELPL2, and the case of dissipative limit (3) DISLIM is depicted in Fig.1. The angular distribution  $d\sigma/d\theta_{\text{cm}}$  for the Xe + Bi reaction is concentrated in a narrow

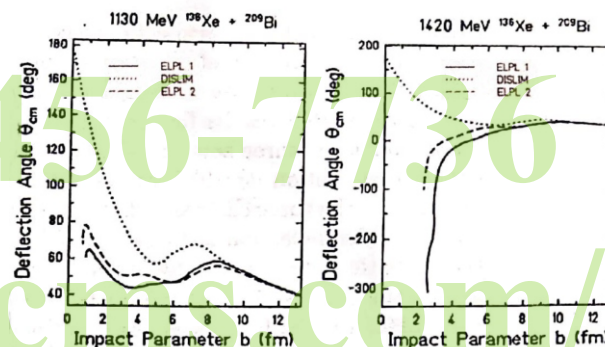


Figure 1: Plot shows the impact parameter  $b$

dependence of the mean deflection angle  $\theta_{cm}$  for three sets of calculations and for two lab energies  $E_{lab} = 1130$  and  $1420$  MeV.

angular range having its maximum at  $\theta_{cm} = 50$  and  $34$  degree respectively slightly forward the quarter point angle  $\theta_{cm} (1)/(4) = 54$  and  $38.0$  degree at bombarding energies  $1130$  and  $1420$  MeV respectively, while towards forward and backward angles the angular distribution drops rapidly. The fast fall of  $d\sigma/d\theta_{cm}$  seems to indicate a negligible contribution of compound nucleus reactions for this system. The cross-section at  $\theta_{cm} = 0.0$  degree is consistent with zero as suggested by the smooth extrapolation. The broadening of the angular distributions  $d\sigma/d\theta_{cm}$  with increasing distance from the projectile suggests an increase in mean interaction time. The angular distribution  $d\sigma/d\theta_{cm}$  for the Xe + Bi reaction productions is sideways peaked (with its maximum near the quarter point angle) but less strongly focused than at the higher bombarding energy. The observed sideways peaked angular distribution can be attributed to a delicate balance between the attractive nuclear forces and repulsive coulomb forces at  $1130$  MeV. Raising the bombarding energy  $E_{lab} = 1420$  MeV is expected to alter this balance of forces and should change the deflection function from a focusing to an orbiting type. There is no indication for orbiting at  $E_{lab} = 1130$  MeV but it has been observed for some what lighter system or higher bombarding energies above the Coulomb energy. However, the angular distribution  $d\sigma/d\theta_{cm}$  measured at a bombarding energy of  $E_{lab} = 1420$  MeV shows a slight forward asymmetry, a result which is not unexpected, in view of the trend established by the measurements at  $E_{lab} = 1130$  MeV. Consideration of the total area under the experimental curve for the cross section  $d\sigma/d\theta_{cm}$  shown in Fig.2 also suggests that the ELPL1 and ELPL2 curves are closer to the measurements, though the peaks for corresponding angular distributions are slightly shifted as compared to the experimental peak. Calculated and measured angular distributions  $d\sigma/d\theta_{cm}$  for three sets of calculations designated as elastoplastic cases (1) ELPL1 and (2) ELPL2, and the case of dissipative limit (3) DISLIM is depicted in Fig. 2

Our calculated results for mean kinetic energy loss  $\Delta E$  as a function of the impact parameter  $b$  for three sets

of calculations are depicted in Fig.3, whereas those for the energy distributions  $d\sigma/d\Delta E$  are depicted in Fig.4. The behavior of the mean energy loss  $\Delta E$  for the three cases shown in Fig.3 can be understood in a similar way as discussed above for the deflection angle. In the DISLIM case due to the absence of repulsion from the diabatic potential the two nuclei come into interaction zone of each other already for relatively larger impact parameters. This causes greater energy loss as compared to the ELPL1 and ELPL2 cases. Also in the case of ELPL1 the loss of angular momentum is at a faster rate and thus for large impact parameters the energy loss is larger than that in the case of ELPL2. The mean kinetic energy loss  $\Delta E$  attains maximum value for very small impact parameters in all three cases irrespective of lab energy. The maximum kinetic energy loss  $\Delta E$  is finally seen to be almost similar in all the three cases but exhibiting different values for different lab energies as for very small impact parameters the total time spent by the two ions within their interaction zone is quite large and the effect of diabatic potential for such creeping motion is negligible. In case of  $E_{lab} = 1130$  and  $1420$  MeV kinetic energy loss maximum attains  $325$  MeV and  $500$  MeV respectively. The maximum amount of energy damped in these heavy-ion collisions is found to be approximately equal to the difference between the initial kinetic energy and the Coulomb barrier for a deformed system estimated from systematic studies of fissioning nuclei. The minimum kinetic energy in the center of mass system is bombarding energy independent and  $150$  MeV smaller than the Coulomb energy of touching spherical fragments indicating the occurrence of elongated dinuclear configuration in the exit channel.

One of the most striking properties of damped heavy-ion reactions is the large amount of kinetic energy of the initial system that may be dissipated into other degrees of freedom during the reaction. The large amount of kinetic energy is dissipated in the Xe + Bi reaction at the two bombarding energies  $1130$  and  $1420$  MeV is illustrated in Fig.4. There is a sharp rise in the energy distribution  $d\sigma/d\Delta E$  at very low energy losses which corresponds to quasi-elastic events contributing very qualitatively about (1)/(6) of the total reaction cross-section. The quasi-elastic contribution joins smoothly a very broad continuous energy loss distribution of damped events extending



over an energy loss range of several hundred MeV. For a given projectile energy  $E_{lab} = 1130$  MeV as energy loss increases energy distribution  $d\sigma/d\Delta E$  increases sharply and becomes maximum at energy loss  $\Delta E = 75$  MeV and then decreases sharply on further increase of energy loss for ELPLI case. In the DISLIM case for bombarding energies 1130 and 1420 MeV the values same of energy losses are larger than the ELPL1 and ELPL2 case for the same value of the energy distribution  $d\sigma/d\Delta E$ . A comparison of the experimental energy distribution data with the calculated results in Fig.4 shows that for smaller en

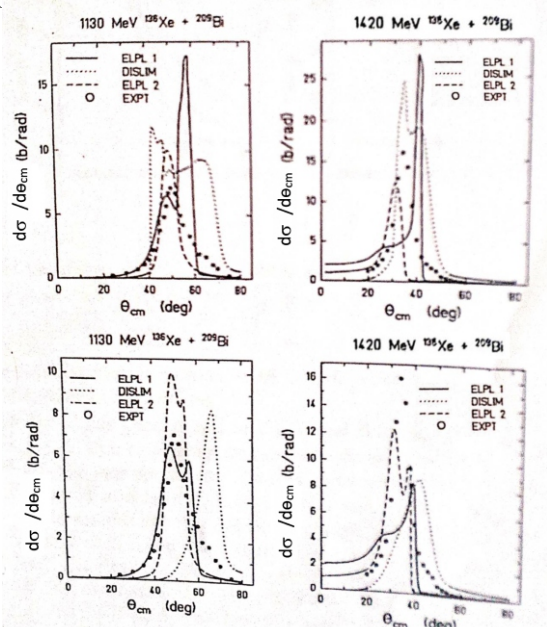


Figure2 :Plot shows the calculated and measured angular distribution  $d\sigma/d\theta_{cm}$  for various values of the mean deflection angle  $\theta_{cm}$  for three sets of calculations and for two lab energies  $E_{lab} = 1130$  and  $1420$  MeV and a comparison with the experimental data.

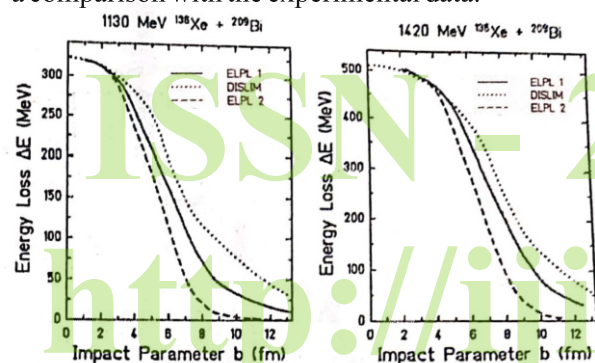


Figure 3: Plot shows the impact parameter  $b$

dependence of the mean kinetic energy loss  $\Delta E$  for three sets of calculations and for two lab energies  $E_{lab} = 1130$  and  $1420$  MeV

$\Delta E$  up to around 50 MeV the DISLIM case leads to cross section  $d\sigma/d\Delta E$  values which are several times larger than the experimental values.

The results for the mean charge  $Z$  has been shown in Fig.5, and the element distribution  $d\sigma/dZ$  are displayed in Fig.6. Due to absence of repulsive diabatic potential in the dissipative limit case DISLIM, the ions spent more time in proximity of each other leading to larger diffusion as compared to other two cases ELPL1 and ELPL2. This is clearly evident in Fig.5 as one observes the variation of mean charge value  $Z$  with decreasing impact parameter. The maximum mean value is attained for the DISLIM case. On increasing bombarding energy the peaks of the curves shifts towards larger mean charge values. It is interesting to note that due to the absence of diabatic repulsion in the angular motion in the case of ELPL1, the exchange of nucleons between nuclei becomes significant only below  $b = 8.0$  fm.

The measured element distributions  $d\sigma/dZ$  are found to be Gaussian at all energies and centered at  $Z=54$  of the projectile. For ELPL1, ELPL2 and DISLIM cases on increasing the projectile  $Z$ , the element distribution rises sharply then becomes maximum at  $Z=55$  and then falls off rapidly on further increase of the projectile  $Z$ . The maximum of the element distribution  $d\sigma/dZ$  remains centered at approximately  $Z=54$  of the projectile for all bombarding energies, while its width increases with bombarding energy. The shape of the  $Z$ -distribution is found to be nearly symmetric. The width of the rather symmetric  $d\sigma/dZ$  distribution is 10  $Z$  units and it extends far out to both sides of the peak. Such a  $Z$ -distribution, which stays centered at the  $Z$  of the initial ions, is characteristic

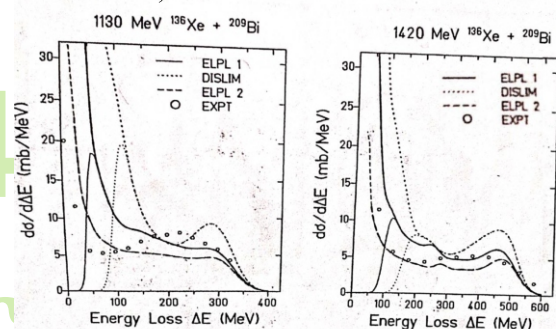


Figure 4: Plot shows the calculated and measured energy distribution  $d\sigma/d\Delta E$  for various values of the



mean energy loss  $\Delta E$  for three sets of calculations and for two lab energies  $E_{\text{lab}} = 1130$  and  $1420$  MeV and a comparison with the experimental data.

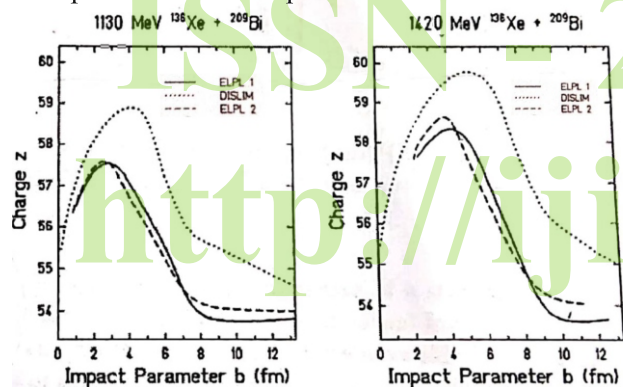


Figure 5: Mean values of the Projectile charge  $Z$  as a Function of impact parameters  $b$  (fm) for elastoplastic cases ELPL1, ELPL2 and the DISLIM case and for two lab energies  $E_{\text{lab}} = 1130$  and  $1420$  MeV.

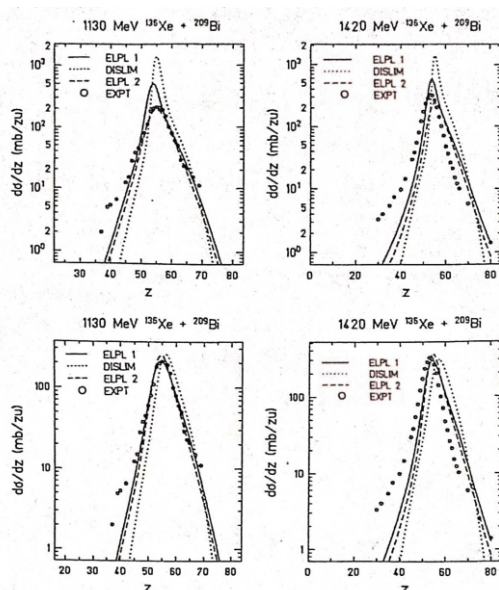


Figure 6: Calculated values of element distribution  $d\sigma/dZ$  (mb)/(Z.u) as a function of projectile charge  $Z$  for ELPL1, ELPL2 and DISLIM cases and a comparison with the experimental data.

of damped reactions between very heavy-ions. It is suggestive of a statistical nucleon exchange process, in which many nucleons may be transferred to and from the projectile with comparable probabilities. The width of the  $Z$ -distribution would then be simply due to the statistical fluctuations of this process. The results for the element distribution  $d\sigma/dZ$  exhibit maximum value approximately 120.0mb/Zunit for all

the three cases at the mean charge  $Z=55$  [one unit above the projectile atomic number]. The  $Z$ -distribution represents  $3/4$  of the total reaction cross-section. Near  $Z=40$  the  $Z$ -distribution is affected by an unknown contribution of sequential fission of the reaction partner. Calculated values of the element distribution  $d\sigma/dZ$  for three sets of calculations and a comparison with the experimental data is depicted in Fig.6.

### 3 Conclusions:

The present communication embodies the results of extensive calculations demonstrating the memory effects (non -Markovian) in dissipative nucleus-nucleus collisions. The calculation have been carried out within the framework of a multi-dimensional FPE, for the reaction  $^{136}\text{Xe}$  (8.31 MeV/u) and (10.4 MeV/u)+ $^{209}\text{Bi}$ . This well known and extensively studied low energy heavy-ion reaction has been used here as a prototype to demonstrate the important features of the theoretical model, especially the effects of the inclusion of diabatic potential (memory effects). It is satisfying to state that the calculations without the inclusion of diabatic potential designed as DISLIM do not exhibit the negative deflection angles observed experimentally in the completely damped region while as the calculations with diabatic potential in the case of ELPL1 and ELPL2 reproduce the negative deflection angles observed in the completely damped region at lab energy  $E_{\text{lab}} = 1420$  MeV. This result supports the validity of the concepts constituting the dissipative diabatic dynamics (DDD). Similarly the results for angular distribution  $d\sigma/d\theta_{\text{cm}}$ , energy distribution  $d\sigma/d\Delta E$  and element distribution  $d\sigma/dZ$  obtained in the ELPLI case are found to provide a satisfactory description of the measured data. The most striking observations made in the present experiment is that the orbiting has been observed at a bombarding energy of  $E_{\text{lab}} = 1420$  MeV. Again the results of DISLIM calculations, in contrast, are not favored by the measurements. A comparison of the experimental data for the element distribution  $d\sigma/dZ$  with calculations enables us to conclude that the ELPL1 description is preferable to that of ELPL2. The calculated results for the mean values of kinetic energy loss  $\Delta E$ , deflection function  $\theta_{\text{cm}}$  and charge  $Z$  for the  $\text{Xe} + \text{Bi}$  system are found to describe rather well the experimental data.

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**References:**

- [1] W. U. Schroder and J. Toke, Report UR-NCUR 2000-002 (2000), and referencetherein.
- [2] H. A. Weidenmüller, Prog. Part. Nucl. Phys. 3 (1980) 49.
- [3] W. Nörenberg and H. A. Weidenmüller, Introduction to the theory of Heavy-Ion Collisions (Springer, Heidelberg, 1980).
- [4] A. Gobbi and W. Nörenberg, in Heavy-Ion Collisions, ed. R. Bock, vol. 2 (North-Holland, Amsterdam, 1980).
- [5] W. Nörenberg, Nucl. Phys. A409 (1983) 191; Nucl. Phys. A400 (1983) 275.
- [6] J.W. Negele, Rev. Mod. Phys. 54 (1982) 913; NATO advance Study Institute of Theoretical Methods in Medium Energy and Heavy-Ion Physics, ed. K.W. McVoy and W.A. Friedman (Plenum, New York, 1978).
- [7] W. Nörenberg, Phys. Lett. 52B (1974) 280; Z. Phys. A274 (1975) 241 and A276 (1976) 84.
- [8] K.H. Schmidt and W. Morawek, Rep. Prog. Phys. 54 (1991) 949
- [9] C.Y. Wong and H.F Tang, Phys. Rev. Lett. 40 (1978) 1070;
- [10] W. Nörenberg, Phys. Lett. 104B (1981) 107
- [11] H.L. Yadav and W. Nörenberg, Phys. Lett. 115B (1982) 180.
- [12] Ayik, Z. Phys. A309 (1982) 121.
- [13] W. Cassing and W. Nörenberg, Nucl. Phys. A401 (1983) 467 and A434 (1984) 558.
- [14] H. Hofman and P. J. Siemens, Nucl. Phys. A257 (1976) 162 and A275 (1977) 464.  
H. Orland and R. Schaeffer, Z. Phys. A290 (1979) 191;  
S. Ayik, Z. Phys. A298 (1980) 83, Nucl. Phys. A370 (1981) 317;
- [15] E. Werner, H. S. Wio, H. Hofmann and K. Pomorski, Z. Phys. A299 (1981) 231.
- [16] S. Pal and N. K. Ganguli, Nucl. Phys. A370 (1981) 175.
- [17] W. Nörenberg and C. Riedel, Z. Phys. A290 (1979) 335.
- [18] K.C Agarwal, G. Saxena. D. Singh & H.L. Yadav, J. Rajasthan Acad. Phys. Sci. 9 (2010) 77.