

STUDY OF CERTAIN SPECIAL TRANSFORMATIONS OF POLY-BASIC ANALOGUE OF SRIVASTAVA - DAoust'S FUNCTION

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Abstract :

A brief study of certain poly-basic hypergeometric functions of srivastava-Daoust has been made by the application of fractional q-derivatives.

Key Words : Hypergeometric functions, Fractional q-derivatives.

Introduction and Definitions

In recent papers Denis⁽¹⁾ and saxena and gupta⁽²⁾ have obtained certain transformations of poly-basic hypergeometric function of srivastava and Daoust⁽³⁾. In this paper we have made a brief study on certain transformations of poly-basic hypergeometric function of srivastava and Daoust of two variables.

The fractional q-derivative of a product of two functions is defined as (cf agarwal⁽⁴⁾).

$$D_q^\lambda(UV) = \sum_{n \geq 0} \frac{(-1)^n q^{n(n+1)/2} (q^{-\lambda}; q)_n}{(q; q)_n} D_q^{\lambda-n}[U(xq^n)] D_q^n(V) \quad \dots(1)$$

Valid for $|x| < R$, $[R = \min(R_1, R_2)]$

where $U(x) = \sum_{r \geq 0} a_r x^r, |x| < R_1,$

$V(x) = \sum_{r \geq 0} b_r x^r, |x| < R_2,$

and also

$$D_q^\alpha x^{u-1} = (1-q)^\alpha \prod \left[\begin{matrix} uq^{-\alpha} \\ u \end{matrix} \right] x^{u-\alpha-1} \quad \dots(2)$$

For $|q| < 1$, the q-shifted factorials are defined by

$$(a; q)_n = \begin{cases} 1, & \text{if } n = 0 \\ (1-a)(1-aq)\dots(1-aq^{n-1}), & \text{if } n = 1, 2, \dots \end{cases}$$

with $(a; q)_\infty = \prod_{n=0}^{\infty} (1-aq^n)$

Further $\prod \left[\begin{matrix} a \\ b \end{matrix} \right]$ stands for $\prod_{n=0}^{\infty} \frac{(1-aq^n)}{1-bq^n}$

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A poly-basic analogue of srivastava and Daoust's⁽³⁾ function is defined as

$$\begin{aligned} & \phi \begin{matrix} A : B + E \\ C : D + F \end{matrix} \left(\begin{matrix} p : [(a; \theta), q; [(b; \alpha)], t : [(e; \nu)] \\ p : [(c; \delta)], q : [(d; \beta)], t : [(f; \epsilon)] \end{matrix} ; x \right) \\ &= \sum_{n=0}^{\infty} \frac{\prod_{i=1}^A (a_i; p)_{n\theta_i} \prod_{i=1}^B (b_i; q)_{n\alpha_i} \prod_{i=1}^E (e_i; t)_{n\nu_i} x^n}{\prod_{i=1}^C (c_i; p)_{n\delta_i} \prod_{i=1}^D (d_i; q)_{n\beta_i} \prod_{i=1}^F (f_i; t)_{n\epsilon_i}} \quad \dots(3) \end{aligned}$$

Where $|q| < 1$, $|p|$, $|t|$, $|x| < 1$, the argument x , the complex parameters

$$\begin{cases} a_i, i = 1, \dots, A; b_i, i = 1, \dots, B; e_i, i = 1, \dots, E \\ c_i, i = 1, \dots, C; d_i, i = 1, \dots, D; f_i, i = 1, \dots, F \end{cases} \quad \dots(4)$$

and the non-negative real coefficients

$$\begin{cases} \theta_i, i = 1, \dots, A; \alpha_i, i = 1, \dots, B; \nu_i, i = 1, \dots, E \\ \delta_i, i = 1, \dots, C; \beta_i, i = 1, \dots, D; \epsilon_i, i = 1, \dots, F \end{cases} \quad \dots(5)$$

being so constrained that the multiple series (3) converge where the parameters of the type (a.) stand for the sequence of parameters a_1, \dots, a_r and will be denoted by (a), when $r = A$. It is evident that (3) defines a uni-basic hypergeometric function for $p = q = t$.

The basic analogue of srivastava and Daoust's (3) function of two variables, which is a special case for $n = 2$ of the corresponding n -variable definition due to srivastava(5)

$$\begin{aligned} & \phi \begin{matrix} A : B, B' \\ C : D, D' \end{matrix} \left(\begin{matrix} [(a; \theta, \phi)]; [(b; \alpha)]; [(b'; \alpha')] \\ [(c; \delta, \epsilon)]; [(d; \beta)]; [(d'; \beta')] \end{matrix} ; q; x, y \right) \\ &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\prod_{i=1}^A (a_i, q)_{m\theta_i + n\phi_i} \prod_{i=1}^B (b_i, q)_{m\alpha_i} \prod_{i=1}^{B'} (b'_i, q)_{n\alpha'_i}}{\prod_{i=1}^C (c_i, q)_{m\delta_i + n\epsilon_i} \prod_{i=1}^D (d_i, q)_{m\beta_i} \prod_{i=1}^{D'} (d'_i, q)_{n\beta'_i}} \frac{x^m y^n}{(q; q)_m (q; q)_n} \\ & \quad |x|, |y|, |q| < 1 \quad \dots(6) \end{aligned}$$

In what follows, the other notations will carry their usual meaning.

TRANSFORMATIONS :

In this section we establish the following transformation of poly-basic analogue of hypergeometric functions :

$$\begin{aligned} &= \phi \begin{matrix} A : C + K \\ B : D + L \end{matrix} \left(\begin{matrix} p : [(a; \theta)], s : [(c; \delta)], r : [(k; n)] \\ p : [(b; \alpha)], s : [(d; \beta)], r : [(l; \psi)] \end{matrix} ; y, \eta \right) \\ & \cdot \phi \begin{matrix} G : E + M \\ H : F + N \end{matrix} \left(\begin{matrix} p : [(g; \zeta)], q : [(e; \gamma)], t : [(m; \phi)] \\ p : [(h; \rho)], q : [(f; \epsilon)], t : [(n; \Omega)] \end{matrix} ; z, \xi \right) \\ &= \phi \begin{matrix} A : E + M \\ B : F + N \end{matrix} \left(\begin{matrix} p : [(a; \theta)], q : [(e; \gamma)], t : [(m; \phi)] \\ p : [(b; \alpha)], q : [(f; \epsilon)], t : [(n; \Omega)] \end{matrix} ; y, \xi \right) \end{aligned}$$

$$\cdot \phi \begin{matrix} G : C + K \\ H : D + L \end{matrix} \left(\begin{matrix} p : [(g; \varsigma)], s : [(c; \delta)], r : [(k; v)] \\ p : [(h; \rho)], s : [(d; \beta)], r : [(l; \psi)] \end{matrix}; z^n \right) \dots (7)$$

To prove (7), we move use of (1) and (2), by taking

$$U(x) = x^{\gamma-1} G \phi H \left(\begin{matrix} [(g; \varsigma)] \\ [(h; \rho)] \end{matrix}; p : xz \right)$$

and $V(x) = A \phi B \left(\begin{matrix} [(a; \theta)] \\ [(b; \alpha)] \end{matrix}; p : xy \right)$

to obtain

$$\sum_{n \geq 0} \frac{(-1)^n p^{n(n+1)/2} (p^{-\lambda}; p)_n p^{n(\gamma-1)} x^n y^n \prod_{i=1}^n (q_i; p)_{n\theta_i}}{(p; p)_n (\gamma p^{-\lambda}; p)_n \prod_{i=1}^B (b_i; p)_{n\alpha_i}} \cdot {}_{G+1} \phi H + 1 \left(\begin{matrix} [(g; \varsigma)], \gamma \\ [(h; \rho)], \gamma p^{-\lambda} \end{matrix}; q; xz p^n \right) \cdot A \phi B \left(\begin{matrix} [(a) p^n; \theta] \\ [(b) p^n; \alpha] \end{matrix}; p; xy \right)$$

$$= \phi \begin{matrix} 1 : G; A \\ 1 : H; B \end{matrix} \left(\begin{matrix} \gamma : [(g; \varsigma)]; [(a; \theta)]; \\ \gamma p^{-\lambda} : [(h; \rho)]; [(b; \alpha)]; \end{matrix} q; xz; xy \right) \dots (8)$$

valid for $|xz| < 1, |xy| < 1, |p| < 1$.

Now if we take

$$U(x) = x^{\gamma-1} A \phi B \left(\begin{matrix} [(a; \theta)]; \\ [(b; \alpha)]; \end{matrix} p; xy \right)$$

and $V(x) = G \phi H \left(\begin{matrix} [(g; \varsigma)]; \\ [(h; \rho)]; \end{matrix} q; xz \right)$

then in view of (1) and (2), we obtain

$$\sum_{n \geq 0} \frac{(-1)^n p^{n(n+1)/2} (p^{-\lambda}; p)_n p^{n(\gamma-1)} x^n z^n \prod_{i=1}^G (g_i; p)_{n\varsigma_i}}{(p; p)_n (\gamma p^{-\lambda}; p)_n \prod_{i=1}^H (h_i; p)_{n\rho_i}} \cdot A + 1 \phi B + 1 \left(\begin{matrix} [(a; \theta)], \gamma; \\ [(b; \alpha)], \gamma p^{-\lambda-n}; \end{matrix} q; xyp^n \right) \cdot G \phi H \left(\begin{matrix} [(g) p^n; \varsigma]; \\ [(h) p^n; \rho]; \end{matrix} p; xz \right)$$

$$\phi \begin{matrix} 1 : G; A \\ 1 : H; B \end{matrix} \left(\begin{matrix} \gamma : [(g; \varsigma)]; [(a; \theta)]; \\ \gamma p^{-n} : [(h; \rho)]; [(b; \alpha)]; \end{matrix} q; xz; xy \right) \dots (9)$$

valid for $|xy|, |xz|, |q| < 1$

It is evident that right hand sides of (8) and (9) are identical and hence by an appeal to analytic continuation, we find that

$$\sum_{n \geq 0} \frac{(-1)^n b^{n(n+1)/2} (p^{-\lambda}; p)_n p^{n(\gamma-1)} x^n y^n \prod_{i=1}^A (a_i; p)_{n\theta_i}}{(p; p)_n (\gamma p^{-\lambda}; p)_n \prod_{i=1}^B (b_i; p)_{n\alpha_i}} \cdot G+1^\phi H+1 \left(\begin{matrix} [(g; \zeta)], \gamma; \\ [(h; \rho)], \gamma p^{-\lambda}; \end{matrix} \middle| p; xz p^n \right) A^\phi B \left(\begin{matrix} [(a) p^n; \theta]; \\ [(b) p^n; \alpha]; \end{matrix} \middle| p; xy \right)$$

$$= \sum_{n \geq 0} \frac{(-1)^n p^{n(n+1)/2} (p^{-\lambda}; p)_n p^{n(\gamma-1)} x^n z^n \prod_{i=1}^A (q_i; q)_{n\theta_i}}{(p; p)_n (\gamma p^{-\lambda}; p)_n \prod_{i=1}^B (b_i; p)_{n\alpha_i}} \cdot G+1^\phi H+1 \left(\begin{matrix} [(g; \zeta)], \gamma; \\ [(h; \rho)], \gamma p^{-\lambda}; \end{matrix} \middle| p; xyp^n \right) A^\phi B \left(\begin{matrix} [(a) p^n; \theta]; \\ [(b) p^n; \alpha]; \end{matrix} \middle| q; xy \right)$$

$$= \sum_{n \geq 0} \frac{(-1)^n p^{n(n+1)/2} (p^{-\lambda}; \phi)_n p^{n(\gamma-1)} x^n z^n \prod_{i=1}^G (g_i; p)_{n\zeta_i}}{(p; p)_n (\gamma p^{-\lambda}; p)_n \prod_{i=1}^H (h_i; p)_{n\rho_i}} \cdot A+1^\phi B+1 \left(\begin{matrix} [(a; \theta)], \gamma; \\ [(b; \alpha)], \gamma p^{-\lambda}; \end{matrix} \middle| p; xyp^n \right) G^\phi H \left(\begin{matrix} [(g) p^n; \zeta]; \\ [(h) p^n; \rho]; \end{matrix} \middle| p; xz \right) \dots(10)$$

whenever both sides exists

Now comparing the coefficients of

$$\frac{(-1)^n p^{n(n+1)/2} (p^{-\lambda}, p)_n p^{n(\gamma-1)} (\gamma; p)_m}{(p; p)_n (\gamma p^{-\lambda}; p)_{n+m}}$$

on both sides of (10), we get after some simplification,

$$\frac{\prod_{i=1}^A (a_i; p)_{n\theta_i} \prod_{i=1}^G (g_i; p)_{m\zeta_i} y^n z^m}{\prod_{i=1}^B (b_i; p)_{n\alpha_i} \prod_{i=1}^H (h_i; p)_{m\rho_i}} A^\phi B \left(\begin{matrix} [(a) p^n; \theta]; \\ [(b) p^n; \alpha]; \end{matrix} \middle| p; xy \right)$$

$$= \frac{\prod_{i=1}^G (g_i; p)_{n\zeta_i} \prod_{i=1}^A (a_i; p)_{m\theta_i} z^n y^m}{\prod_{i=1}^B (h_i; p)_{n\rho_i} \prod_{i=1}^B (b_i; p)_{m\alpha_i}} G^\phi H \left(\begin{matrix} [(g) p^n; \zeta]; \\ [(h) p^n; \rho]; \end{matrix} \middle| p; xy \right) \dots(11)$$

Now multiplying both the sides by

$$\frac{\prod_{i=1}^E (e_i; q)_{m\gamma_i} \prod_{i=1}^M (m_i; t)_{m\phi_i}}{\prod_{i=1}^F (f_i; q)_{m\epsilon_i} \prod_{i=1}^N (n_i; t)_{m\Omega_i}} \xi^m, \text{ we get}$$

$$\frac{\prod_{i=1}^A (a_i; p)_{n\theta_i} \prod_{i=1}^G (g_i; p)_{m\zeta_i} \prod_{i=1}^E (e_i; q)_{m\gamma_i} \prod_{i=1}^M (m_i; t)_{m\phi_i}}{\prod_{i=1}^B (b_i; p)_{n\alpha_i} \prod_{i=1}^H (h_i; p)_{m\rho_i} \prod_{i=1}^F (f_i; q)_{m\epsilon_i} \prod_{i=1}^N (n_i; t)_{m\Omega_i}} y^n z^m \xi^m$$

$$\cdot A^{\phi} B \left(\begin{matrix} [(a)p^n; \theta]; \\ [(b)p^n; \alpha]; \end{matrix} \begin{matrix} p; xy \end{matrix} \right)$$

$$= \frac{\prod_{i=1}^G (g_i; p)_{n\zeta_i} \prod_{i=1}^A (a_i; p)_{m\theta_i} \prod_{i=1}^E (e_i; q)_{m\gamma_i} \prod_{i=1}^M (m_i; t)_{m\phi_i}}{\prod_{i=1}^H (h_i; p)_{n\rho_i} \prod_{i=1}^B (b_i; p)_{m\alpha_i} \prod_{i=1}^F (f_i; q)_{m\epsilon_i} \prod_{i=1}^N (n_i; t)_{m\Omega_i}} z^n y^m \xi^m$$

$$\cdot G^{\phi} H \left(\begin{matrix} [(g)p^n; \zeta]; \\ [(h)p^n; \rho]; \end{matrix} \begin{matrix} p; xz \end{matrix} \right) \dots(12)$$

Now summing from $m = 0$ to ∞ , we get

$$\frac{\prod_{i=1}^A (a_i; p)_{n\theta_i}}{\prod_{i=1}^B (b_i; p)_{n\alpha_i}} A^{\phi} B \left(\begin{matrix} [(a)p^n; \theta]; \\ [(b)p^n; \alpha]; \end{matrix} \begin{matrix} p; xy \end{matrix} \right) \cdot y^n$$

$$\cdot \phi \begin{matrix} G : E + M \\ H : F + N \end{matrix} \left(\begin{matrix} p : [(g; \zeta)], q : [(e; \gamma)], t : [(m; \phi)] \\ p : [(h; \rho)], q : [(f; \epsilon)], t : [(n; \Omega)] \end{matrix} ; z\xi \right)$$

$$= \frac{\prod_{i=1}^G (g_i; p)_{n\zeta_i}}{\prod_{i=1}^H (h_i; p)_{n\rho_i}} G^{\phi} H \left(\begin{matrix} [(g)p^n; \zeta]; \\ [(h)p^n; \rho]; \end{matrix} \begin{matrix} p; xz \end{matrix} \right) \cdot z^n$$

$$\cdot \phi \begin{matrix} A : E + M \\ B : F + N \end{matrix} \left(\begin{matrix} p : [(a; \theta)], q : [(e; \gamma)], t : [(m; \phi)]; \\ p : [(b; \alpha)], q : [(f; \epsilon)], t : [(n; \Omega)]; \end{matrix} y\xi \right) \dots(13)$$

Again comparing the coefficients of $\frac{x^R}{(p : p)^R}$ on both sides of (13) and then putting $n + R = m$, we get

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$$\frac{\prod_{i=1}^A ((a_i; p)_{m\alpha_i})}{\prod_{i=1}^B (b_i; p)_{m\alpha_i}} y^m \phi \begin{matrix} G : E + M \\ H : F + N \end{matrix} \left(\begin{matrix} p : [(g; \varsigma)], q : [(e; \gamma)], t : [(m; \phi)]; \\ p : [(h; \rho)], q : [(f; \epsilon)], t : [(n; \Omega)]; \end{matrix} z\xi \right)$$

$$= \frac{\prod_{i=1}^G (g_i; p)_{m\delta_i}}{\prod_{i=1}^H (h_i; p)_{m\beta_i}} z^m \phi \begin{matrix} A : E + M \\ B : F + N \end{matrix} \left(\begin{matrix} p : [(a; \theta)], q : [(e; \gamma)], t : [(m; \phi)]; \\ p : [(b; \alpha)], q : [(f; \epsilon)], t : [(n; \Omega)]; \end{matrix} y\xi \right) \dots (14)$$

Now multiplying both the sides of (14) by

$$\frac{\prod_{i=1}^C (c_i; \varsigma)_{m\delta_i} \prod_{i=1}^K (k_i; r)_{m\nu_i}}{\prod_{i=1}^D (d_i; \varsigma)_{m\beta_i} \prod_{i=1}^L (l_i; r)_{m\psi_i}} \cdot \eta^m$$

and then summing both the sides w.r.t. m from m = 0 to ∞ we get the required result.

It is evident that, following the above method, one can easily establish transformations involving multivariable basic hypergeometric functions of several variables of srivastava, when the complex constants are each equal to unity.

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