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Study of Dissipative Heavy-Ion Collisions

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Abstract: Heavy-ion reactions with energies of a few MeV per nucleon above the Coulomb barrier are well known to exhibit dissipative processes [1,2] in nuclei. Such dissipative collisions, also called Deeply Inelastic Collisions (DIC), are characterized by dissipation of a large amount of kinetic energy and angular momentum of the relative motion into intrinsic excitations, as well as by the diffusion of nucleons between the two colliding" heavy-ions [3,4]. These reactions cover the entire range between direct reactions and compound nucleus formation. Whereas only a few degrees of freedom are involved in a direct reaction, all degrees of freedom participate in compound nucleus formation. Thus dissipative reactions carry information regarding the relaxation processes leading simple nuclear states to more complex configurations.

The interaction times of two heavy-ions in dissipative or deeply in-elastic collisions (DIC) are typically of the order 10^{-21}s. Although this time is not much larger than the typical time of a direct reaction 10^{-22} s, DIC are found to be characteristically different from the direct reactions as the former exhibit dissipative processes and involve equilibration phenomena.

Extensive studies in dissipative heavy-ion collisions have opened up new possibilities to widen my understanding of nuclear properties, especially the behavior of nuclear matter associated with the dissipation of energy, angular momentum and mass. Moreover. Investigations of such reactions provide valuable information with regard to the relaxation times associated with different degrees of freedom. In this paper, the heavy-ion reaction $^{86}_{36}Kr(8.18\ MeV/u) +\ ^{166}_{68}Er$ has been used as a prototype for calculation of interaction time & deflection function.

Keywords: Heavy-ion Collisions, DIC, Interaction time & Deflection function.

Introduction:

Main Experimental Results on Dissipative Collisions:

Experimental studies of the dissipative heavy-ion collisions have been extensively carried out during the last several decades [5-10]. The important features of these experimental results are:

i. The identity of projectile and target, respectively, is essentially preserved, although a considerable amount of mass ($\Delta A \leq 30$) can be transferred during the collisions.

ii. The incident energy is typically 2-10 MeV per nucleon at the Coulomb barrier. A large fraction of this energy is dissipated into intrinsic excitation energy of either fragment, leading to energy losses of typically several 100 MeV within less than 10^{-21} s.

iii. Up to 40-50 units of $\hbar$ of angular momentum of relative motion are converted into intrinsic spin of either fragment.

iv. The angular distribution $d\sigma/d\theta$ is strongly non-isotropic which implies that the nuclear interaction time $\approx (10^{-22} - 10^{-20})$ s is significantly smaller than the time needed for a complete rotation of the composite system $(\tau_{rot} > 10^{-20})s$.

v. Dissipative collisions cover the entire range between direct reactions and compound nucleus formation. Their share in the total cross-section increases with increasing bombarding energy, and the masses of the colliding nuclei. For heavy-nuclei, typical values for the dissipative collisions cross-sections are of the order of several barns.

Distant, Grazing and Close Collisions:

A very schematic picture of the various processes has been given in Fig. 1. According to this classical picture, I distinguish three qualitatively different types of collisions [8,11]. They are determined by the impact parameter b or the corresponding relative angular momentum $l = bk_\infty$ where k_∞ denotes the initial (asymptotic) relative wave number.

I define the grazing or critical impact parameter denoted by b_{gr} or b_{crit} for the trajectory which leads just to a considerable nuclear interaction between projectile and target. Correspondingly, I define an interaction radius R which is given by the closest distance of approach between the centers of the nuclei on the grazing trajectory. For grazing collisions ($b \approx b_{gr}$) the duration time τ_{int} of nuclear interaction (nuclear contact time) between projectile and target is small by definition. Therefore, I expect reactions to dominate where only a few internal degrees of freedom of projectile and target are involved. These are called the direct reactions. For impact parameters which are considerably larger than b_{gr}, the nuclear interaction is negligible. The trajectories of these distant collisions are completely determined by the Coulomb interaction. Only excitations induced by the mutual Coulomb interaction between the nuclei (Coulomb excitations) can occur along these Coulomb trajectories.

For impact parameters considerably smaller than b_{gr} I expect a strong disturbance of projectile and target by their mutual nuclear interaction. Practically all nucleonic degrees of freedom are involved in these close collisions. I define the nuclear interaction time τ_{int} as the time interval during which two interacting nuclei are within the sphere given by the interaction radius R_{int}. This interaction time is considerably larger than those for direct reactions. Such a close collision can lead to a completely equilibrated intermediate stage, the compound nucleus. This compound nucleus has by definition a lifetime which is large compared to the time-of-flight of the projectile through the interaction region, sphere of radius R. The compound nucleus formation is a possible but not a necessary intermediate stage in a close collision. Especially, in collisions between rather heavy nuclei a new type of reaction has been observed which is referred to as dissipative collisions. These dissipative collisions correspond to a band of impact parameters larger than the critical impact parameter b_{crit}, where compound nucleus formation begins to take place, and smaller than the grazing impact parameter b_{gr}, where direct reactions occur. Thus, deeply inelastic collision refer to all reactions where multi-step processes but no compound nucleus formation occur. The dissipative collisions are characterized by a strongly non-isotropic angular distribution and, therefore, by short interaction times of the order of $10^{-22} - 10^{-20}$ s. Such reactions are further marked by strong dissipation of relative kinetic energy and angular momentum into extrinsic excitations, and by transfer of a considerable amount of mass between projectile and target. Further, there is a smooth transition between direct processes and dissipative collisions.

Description of The Model:

Generally, the nuclear interaction time for a heavy-ion reaction may be obtained by performing a classical or semi-classical dynamical calculation [12-14] with conservative and friction forces, whereby one solves the complex equations for the mean values and fluctuations of the quantities described by the collective variables. These collective variables are usually the distance between the centres of mass, relative momentum, angle of deflection, angular momentum of the relative motion, mass asymmetry, and the deformation of the projectile and target nuclei. These calculations, however, require involved computational efforts and moreover, suffer from the uncertainties of the nucleus-nucleus potentials. I, therefore, have used here a simpler and direct method for obtaining the interaction time for the colliding heavy-ions from the experimental data. In this approach the connection between impact parameter and scattering angle is obtained by constructing a deflection function from the experimental angular distribution, whereas energy and angular momentum dissipation are taken into account in the calculation of the mean interaction time [15-16].
The classical model for the calculation of the mean interaction time is illustrated in Fig. 2. The figure shows schematically a trajectory for the collision of a projectile nucleus of radius R_1 with a target nucleus of radius R_2 the projectile being incident with initial centre of mass energy E_i, relative angular momentum l_i and impact parameter b_i. The range of the interaction R has been indicated by a dashed circle around the target nucleus. After the interaction the asymptotic values of the energy, relative angular momentum and the impact parameter of the outgoing nucleus have the values E_f, l_f and b_f, respectively. The corresponding deflection angle for the collision has also been shown. The scattering angles θ_i and θ_f correspond to the Coulomb trajectories in the entrance and exit channels, whereas $\Delta\theta$ denotes the rotation of the composite system during the entire interaction time $\tau_{int}\ (l_i)$.
The deflection function Θ is uniquely determined by the incident centre of mass energy E_i and the impact parameter b_i or, alternatively, relative angular momentum l_i. The projectile moves on a Coulomb trajectory up to the interaction radius R, which in the classical approximation, is related to the reaction cross section through

$$\sigma_R = \pi b_{gr}^2 = \pi R^2 \left(l - V_B/E_i\right) \qquad \dots (1.1)$$

In which the Coulomb potential at the interaction radius R is given by

$$V_B = Z_1 Z_2 e^2/R \equiv \alpha/R \qquad \dots (1.2)$$

Where I introduce $\alpha = Z_l Z_2 e^2$

For calculating the interaction time as a function of the impact parameter, I make the following assumptions:

(i) The two nuclei form a rotating composite system at close contact

$$R_o = 1.2\,(A_l^{l/3} + A_2^{l/3}) \qquad \dots (1.3)$$

(ii) The radial part of the relative kinetic energy is completely dissipated during the interaction time. Thus, the final energy is given by

$$E_f = V_B + \frac{l_f^2 h^2}{2J_{rel}} \qquad \dots (1.4)$$

Where l_f is the final angular momentum.

(iii) The relative angular momentum is transferred gradually into intrinsic angular momentum of the nuclei. I assume as suggested by the calculations of transport coefficients that the rate of angular momentum dissipation is proportional to the available relative angular momentum.

$$\frac{\partial l}{\partial t} = (l/\tau_{ang})\,[l(t) - l_{st}] \qquad \dots (1.5)$$

Here τ_{ang} denotes the relaxation time for angular momentum dissipation and l_{st} is the sticking value for the relative angular momentum. Sticking means the fragments jointly rotate like a rigid body with a common velocity together with angular momentum conservation. This gives

$$l_{st} = (J_{rel}^o / J_{tot}^o) l_i$$

$$\omega_{st} = \hbar\, l_{st}/J_{rel}^o \qquad \dots (1.6)$$

Integration of eqn. (1.5) yields

$$l(t) = l_{st} + (l_i - l_{st}) \exp(-t/\tau_{ang}) \qquad \dots (1.7)$$

Which may also be written in the form

$$l(l_i, \Delta\theta) = l_{st} + (l_i - l_{st}) \exp[-(\Delta\theta/\omega_{st})/\tau_{ang}] \qquad \dots (1.8)$$

The relaxation time τ_{ang} has been determined from a fit to experimental energy spectra for the angular momentum and energy dissipation [16-17]. At this point it should be emphasized that deformation of the colliding nuclei during their interaction is expected to play an important role and should be included as such for a complete treatment. However, for the sake of simplicity, deformation degree of freedom is not considered here.

For the relative and total moments of inertia, I take the rigid body values

$$J_{rel}^0 = \mu R_0^2\,, \qquad J_{rel} = \mu R^2$$

$$J_{tot}^0 = J_{rel}^0 + \frac{2}{5}\, m_l R_l^2 + \frac{2}{3} m_2 R_2^2 \qquad \dots (1.9)$$

Where μ, m_1. m_2, R_1 and R_2 are, respectively, the reduced mass, the masses and radii of two fragments. R_o denotes the contact radius whereas R stands for the interaction radius, defined through the relations (1.3), (1.1) and respectively.

The angle Δ θ of rotation of the composite system is given by

$$\Delta\theta = \pi - \theta_i - \theta_f - \Theta \qquad \dots (1.10)$$

for each impact parameter b_i (or initial angular momentum). Here the Coulomb angles θ_i and θ_f are determined by the Coulomb trajectories in entrance and exit channels with the corresponding values of energy E and impact parameter b through the relation.

$$\theta = sin^{-1}\left[(2b/R+\varepsilon)/\sqrt{4}+\varepsilon^2\right] - sin^{-1}\left[l/\sqrt{(2/\varepsilon)^2}+l\right] \quad \ldots (1.11)$$

Where

$$\varepsilon = \frac{\alpha}{Eb}; \quad b = \hbar\ l/\sqrt{2\mu E} \quad ; \quad \alpha = Z_l Z_2 e^2 \quad \ldots (1.12)$$

For θ_i I use the given initial energy E_i and the impact parameter b_i (or equivalently angular momentum l_i). Similarly, for θ_f, I employ the values E_f and b_f (or l_f) given by eqns. (1.4) and (1.8), respectively.

The relation between $\Delta\ \theta$, the angle of rotation, and the interaction time τ_{int} is defined by the integral

$$\Delta\theta = \int_0^{\tau_{int}} dt \left(\frac{d\theta}{dt}\right) = \int_0^{\tau_{int}} dt \left[\frac{\hbar\ l(t)}{\mathfrak{I}_{rel}\ (t)}\right] \quad \ldots (1.13)$$

A straight forward integration of (1.13) with l(t) given by (1.8) gives

$$\Delta\theta = \frac{\hbar l_{st}}{\mathfrak{I}_{rel}^{o}}\left[\tau_{int} + \tau_{ang}(l_i - l_f)/l_{st}\right] \quad \ldots (1.14)$$

This finally yields for the interaction time

$$\tau_{int}(l_i, \Delta\theta) = \Delta\theta[\mathfrak{I}_{rel}^{o}/(\hbar l_{st}] - \tau_{ang}(l_i - l_f)/l_{st} \quad \ldots (1.15)$$

Determination of Deflection Function:

The essential part of the model is the determination of the deflection function $\Theta(l_i)$ from the experimental angular distribution $\frac{d\sigma}{d\Omega}$ for given impact parameter b_i, or alternatively for a given initial angular momentum value l_i. For this purpose, I parameterize the deflection function $\Theta(l_i\ \equiv l)$ or $(b_i\ \equiv b)$ [12,13,18].

$$\Theta(b) = \Theta_C(b) + \Theta_N(b) \quad \ldots (1.16)$$

Where $\Theta_C(b)$ denotes the contribution from the Coulomb interaction, and Θ_N accounts for the nuclear interaction effect. The Coulomb deflection function is given by the Coulomb scattering.

$$\Theta_C(b) = 2tan^{-1}[\alpha/(2Eb)] \quad \ldots (1.17)$$

where $\alpha = Z_l Z_2 e^2$. The term describing the deviation from the Coulomb deflection function Θ_C due to the nuclear interaction between the projectile and target nuclei is assumed to have the form [6,10,11]

$$\Theta_N(b) = -\beta\ \Theta_C^{gr}\left(\frac{b}{b_{gr}}\right)(\delta/\beta)^{b/b_{gr}} \quad \ldots (1.18)$$

Here b_{gr} is the grazing impact parameter, and Θ_C^{gr} is the Coulomb deflection angle on the grazing trajectory. From the form of this parameterization, it is evident that $b = b_{gr}$ one obtains $\Theta_N(b = b_{gr}) = -\delta_C^{gr}$, indicating thereby that the deviation from the Coulomb trajectory near the grazing angle is determined by the parameter δ. For large value of β, the deflection function may become negative and hence it describes the nuclear effects for large interaction time. On the other hand, a very small value of β makes the second term Θ_N in eq. (1.16) negligible. Large values of β which correspond to large interaction time τ_{int} occur for large value of incident projectile energy E such that $E/V_B \geq\ 1.7$.

Table 1 Deflection function $\Theta(l_i)$ for different values of initial angular momentum l_i (or corresponding b_i) as obtained by fitting the experimental angular distribution for the heavy-ion reaction ${}^{86}_{36}Kr(8.18MeV/u) +\ {}^{166}_{68}Er$.

Table 1: Some important experimental and calculated quantities for the reaction ${}^{86}_{36}Kr(8.18\ MeV/u) +\ {}^{166}_{68}Er$

1	Incident energy per nucleon E_i/A_1	8.18 MeV/u	6	Coulomb barrier $V_B(R)$	256.5 MeV
2	Grazing impact parameter b_{gr}	9.18 fm	7	Ratio $E_{cm}/V_B(R)$	1.81
3	Grazing angular momentum l_{gr}	325 (ħ)	8	Experimental reaction cross section σ_R	2648 mb
4	Interaction radius R	13.74 fm	9	Maximum value of the experimental total kinetic energy loss for the reaction $(TKEL)_{max}^{expt} = (E_{diss})_{max}^{expt}$	270 MeV
5	Contact radius R_0	11.88 fm	10	Values of parameters for the deflection function β δ	70 0.17

Table 2: Deflection function $\Theta(l_i)$ for different values of initial angular momentum l_i (or corresponding b_i) as obtained by fitting the experimental angular distribution for the heavy-ion reaction ${}^{86}_{36}Kr\left(8.18\frac{MeV}{u}\right) + {}^{166}_{68}Er$.

l(ħ)	b(fm)	$\frac{l}{l_{gr}}$	$\Theta_C(b)$ degree	$\Theta_N(b)$ Degree	$\Theta(b)$ degree
0	0	0	180	0	180
25	0.71	0.08	159.36	-152.5	6.46
50	1.40	0.15	139.25	-192.37	-53.12
75	2.12	0.23	122.16	-181.24	-59.08
100	2.80	0.31	107.19	-152.06	-44.47
125	3.50	0.38	94.25	-120.02	-25.37
150	4.20	0.46	84.30	-90.35	-6.05
175	4.90	0.54	75.15	-66.31	8.44
200	5.60	0.62	68.29	-48.09	20.20
225	6.30	0.70	62.19	-34.17	28.02
250	7.00	0.77	57.01	-24.01	33.00
275	7.80	0.85	52.17	-16.35	35-32
300	8.50	0.92	48.34	-11.22	37.12
325	9.18	1.00	45.01	-8.05	36.56

Table 3: Calculated values of $\tau_{int}\ (l_i)$ for different values of initial angular momentum l_i (in unit ħ) for the reaction ${}^{86}Kr(8.18\ MeV/u)\ +\ {}^{166}Er$. The table also gives the corresponding angular momentum loss and energy loss for each initial angular momentum l_i (ħ).

Initial angular momentum l_i (ħ)	Angular momentum loss l_{diss}(ħ)	Total kinetic energy loss $E_{diss}\ (MeV)$	Interaction time $\tau_{int}(10^{-21}s)$
25	8.33	206.4	22.15
50	16.64	204.8	13.70
75	25.00	198.6	9.05
100	33.33	196.6	5.81
125	41.65	193.94	3.71
150	49.98	188.24	2.34
175	58.31	181.50	1.44
200	42.14	151.42	0.79
225	17.88	140.2	0.41
250	13.38	102.45	0.37
275	11.18	52.40	0.28
300	9.46	49.42	0.24
325	5.50	16.40	0.20

Calculation of Interaction Time τ_{int} (l_i)

The classical interpretation of the Wilczynski plot implies that the trajectories and hence the deflection function $\Theta(l_i)$ are uniquely determined by the initial angular momentum l_i or the corresponding impact parameter b_i. For the reaction $^{86}Kr(8.18\ MeV/u)\ +\ ^{166}Er$ the nuclear interaction time increases with decreasing deflection angle, and hence, the deflection angle serves in this case as a stop watch for providing a measure of the interaction time. In order to calculate interaction time from the deflection function $\Theta(l_i)$, I solve the coupled eqns. (1.4), (1.8), (1.10), (1.11) and (1.15) by iteration. For the first step of the iteration procedure, I start with $l_f = l_{st}$ for the unknown l_f. For this value of l_f, eqn. (1.4) yields the value of E_f. Using these values of l_f and E_f, the Coulomb trajectory for the exit channel is calculated from eqn. (1.11) to obtain θ_f. Similårly θ_i is obtained for the initial values of the energy E_i and angular momentum l_i. These values of θ_i, θ_f and the value of $\Theta(l_i)$ of the deflection function obtained, for the initial angular momentum l_i and incident energy E_i, are used to obtain corresponding angle of rotation $\Delta\theta(l_i, E_i)$ from eqn. (1.10). Then eqn. (1.15) is employed to calculate the interaction time τ_{int} $(l_i, \Delta\theta)$, wherein I use the value $\tau_{ang} = 1.5\ \times 10^{-21}s$ for the angular momentum relaxation time obtained in a fit to the experimentally determined dissipation energy and dissipation angular momentum [12,19]. The interaction time τ_{int} $(l_i, \Delta\theta)$ calculated in this first step of iteration is substituted in eqn. (1.8) to obtain l_f (τ_{int}) This new value of $l_f(\tau_{int})$ thus obtained provides the starting value of l_f and E_f for the second iteration. The process is continued until one obtains a consistent value of τ_{int} $(l_i, \Delta\theta)$ and l_f for the given initial angular momentum l_i or corresponding impact parameter b_i. This completes the calculation of τ_{int} $(l_i, \Delta\theta)$ as a function of l_i, or b_i for the reaction $^{86}Kr(8.18\ MeV/u)\ +\ ^{166}Er$ considered here.

My calculated results for the interaction time τ_{int} (l_i) as a function of initial angular momentum l_i have been shown in Fig 4.

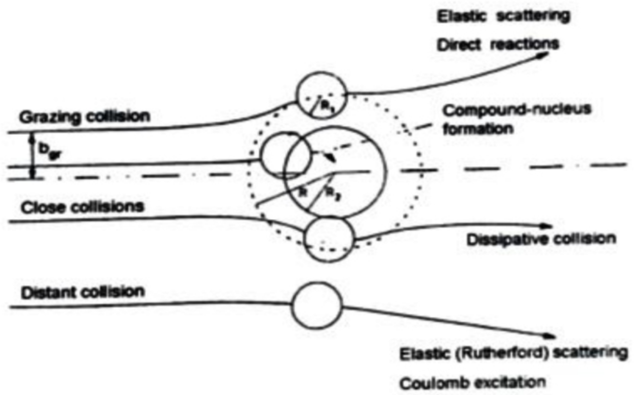

Fig. 1. Distant, grazing and close collisions in the classical picture of heavy-ion collisions.

Fig. 2. Illustration of the classical model for the mean interaction time in dissipative

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heavy-ion collisions. The Coulomb trajectories shown are for the initial impact parameter and final impact parameter b_i = 6fm & b_f=5.1 fm.

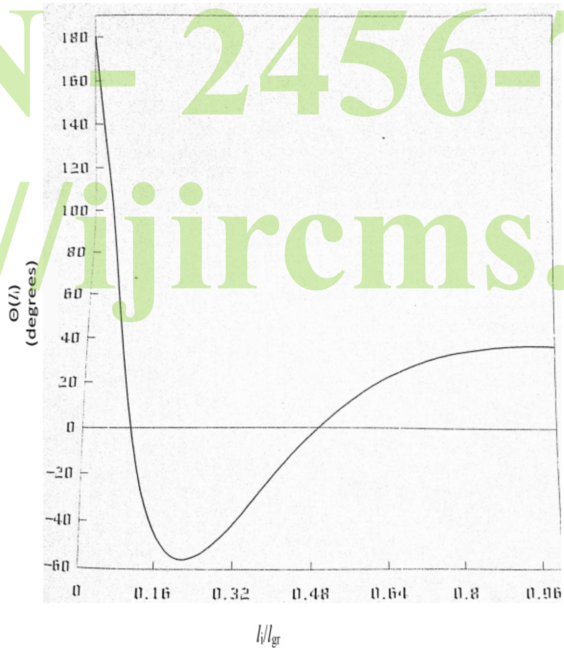

Fig. 3. Deflection function $\Theta(l_i)$ as a function l_i/l_{gr} obtained from a fit of the experimental angular distribution for the reaction ${}^{86}_{36}Kr(8.18\, MeV/u) + {}^{166}_{68}Er$

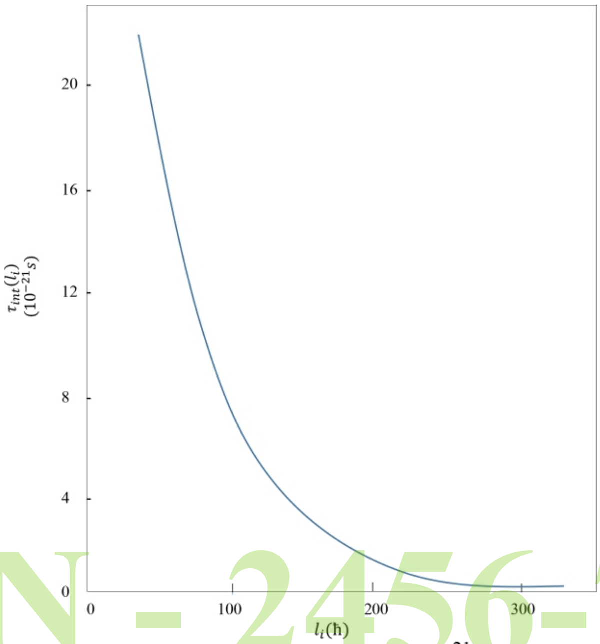

Fig. 4. Calculated interaction time $\tau_{int}(l_i)$ in unit of 10^{-21}s as a function of initial angular momentum $l_i(\hbar)$

Results & Discussion:

In this paper, I have tested a simple classical model for calculation of interaction time and deflection function in ${}^{86}_{36}Kr(8.18\, MeV/u) + {}^{166}_{68}Er$ reaction. In this model, the interaction time for the collision (contact time of the two heavy-ion) is obtained directly from the experimental data for the angular distribution cross-section. Thus, my estimate of interaction time τ_{int} dose not suffer from the uncertainties of the complex dynamical calculations using friction force and the transport coefficients. The calculated results for the interaction time τ_{int} as a function of initial angular momentum l_i shows that the interaction time below the grazing angular momentum $l_{gr} \approx 325\hbar$ first increases slowly, and then rapidly with decreasing initial angular momentum l_i. Measurements for the reaction $^{86}_{36}Kr(8.18\ MeV/u) + {}^{166}_{68}Er$ show that the deflection angle becomes negative for the completely damped components. In the case of smaller impact parameters b for which the interaction time τ_{int} is relatively large, two nuclei lead to large negative deflection angle. For very small impact parameters b, the interpenetration of two nuclei leads even to positive angle of deflection. The calculated results for the interaction time and deflection function for the reaction are consistent with the experimental data and will be improved if deformation degrees of freedom of the heavy-ion are included in this model.

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