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Deteriorating Inventory Model with Linear Demand Pattern Considering Random Deterioration

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Abstract: An inventory model for deteriorating items with linear demand rate is being developed. The depletion of inventory is not due to demand but also due to deterioration, as there are almost all items that undergo deterioration. So rate of deterioration is important to be considered, while analysing the inventory system. Some items undergo deterioration with time as well as with some other factor such as humidity, temperature etc. known as random rate of deterioration.

Keywords: Inventory system, Random deterioration rate, Linear Demand.

1. Introduction:

Inventory models for deteriorating items have being studied by many researchers. First of all Ghare and Schrader were pointed out the effect of decay in inventory management. Decay or deterioration refers to damage, spoilage, vaporization or obsolescence of the items. Almost all items will deteriorates, if stored for extended periods of time. Some items such as metal parts undergoes corrosion and rusting. Spoilage and decay occurs in food products together with electronic components and fashion clothing. Hence deterioration factor has to be given importance while determining the optimal policy of an inventory model.

Recently many researchers are analyzing the effect of deterioration and variation in the demand rate with time taking different deterioration rate such as constant or time dependent or linear function of time or two parameter Weibull function or random deterioration rate. Covert and Philip (1) developed inventory model for variable rate of deterioration by assuming two parameter Weibull distribution function. Some researchers like Deb and Chaudhari(2) and Mandal and Phaujdar(5) developed inventory model with time dependent demand. Further Mandal and De(6) developed an EOQ model taken parabolic demand rate and time varying selling price.

Some authors like Gupta and Vart(4), Sharma and Chaudhary(8) and Su, Tong and Liao(11), who formulated model with demand rate dependent on the initial stock level. Singh et al. developed inventory model for items having linear demand and variable deterioration rate with trade credit. Sharma, Aggarwal and Khurana(7) extended this model and gave an EOQ model for deteriorating items with ramp type demand with Weibull distributed deterioration and shortages are allowed. Also Shikha, Kishan and Rani(9) developed an inventory model of perishable products with displayed stock level dependent demand rate. Singh(10) established the inventory model for perishable items having constant demand with time dependent holding cost.

In the present paper, attempts have been made to develop an inventory model with linear demand and deterioration of items depends on time together with an another factor such as fluctuation of humidity, temperature etc. It would be more reasonable and realistic, if we assume the deterioration function 'θ ' to depend on another parameter 'α', in addition to time '*t*' which ranges over a space 'Γ' and in which a probability density function $\theta_0(\alpha)$ is defined.

2. Assumptions and natations:

An inventory model for deteriorating product for finite time horizon is developed under following assumptions and notations.

(I) A single inventory will be used.
(ii) Lead time is very-very small or zero.
(iii) Shortages are allowed and are completely backlogged .
(iv) The demand rate $D(t) = a + bt, a > 0, b > 0$ at any time 't' i.e. follows linear demand pattern.
(v) Replenishment rate is infinite but size is finite and is assumed to be 'Q'.
(vi) Time horizon is finite.
(vii) The second and higher powers of α are neglected in this analysis.
(viii) There is no repair of deteriorated items occurring during the cycle. Let 'T' be the duration of a cycle.
(ix) Let 'V' is the inventory at t=0.
(x) Let $I(t)$ be the on hand inventory level at any time $t, t \geq 0$.
(xi) Let the deterioration cost per unit item is, the holding cost per unit item is 'C_h'and shortage cost of the system cost per unit is 'C_s'.
(xii) Deterioration function $\theta(t,\alpha)$ is assumed in the form $\theta(t,\alpha) = \theta_0(\alpha)t$, $0 < \theta_0(\alpha) << 1$, $t > 0$.

3. MATHEMATICAL FORMULATION AND ANALYSIS FOR THE SYSTEM

Let 'Q' be the total amount of stock replenished in the beginning of each cycle and after fulfilling backorders, let 'V' be the level of initial inventory. During the period $(0,t_1)$, the inventory level gradually decreases due to market demand and deterioration. At time $t=t_1$, the inventory level reaches to zero and after that shortages occurs in $[t_1,T]$ which are fully backlogged. Only the backlogging items are replaced by the next replenishment. The behaviour of inventory is depicted in the following inventory time diagram.

If $I(t)$ be the on hand inventory at time $t \geq 0$, then at time $t+\Delta t$, the on hand inventory in the interval $[0,t_1]$ will be

$$I(t+\Delta t) = I(t) - \theta_0(t,\alpha)I(t)\Delta t - D(t)\Delta t$$

or, $$I(t+\Delta t) - I(t) + \theta_0(t,\alpha)I(t)\Delta t = -D(t)\Delta t$$

Dividing by Δt and taking limit $\Delta t \to 0$, we get

$$\lim_{\Delta t \to 0}\left(\frac{I(t+\Delta t)-I(t)}{\Delta t}\right) + \theta_0(t,\alpha)I(t) = -D(t)$$

$$\frac{dI(t)}{dt} + \theta_0(t,\alpha)I(t) = -D(t)$$

$$\frac{dI(t)}{dt} + \theta_0(t,\alpha)I(t) = -(a+bt) \quad 0 \leq t \leq t_1 \tag{1}$$

For the next interval $[t_1,T]$, we have

$$I(t+\Delta t) = I(t) - D(t)\Delta t$$

$$\lim_{\Delta t \to 0}\left(\frac{I(t+\Delta t)-I(t)}{\Delta t}\right) = -D(t)$$

$$\frac{dI(t)}{dt} = -(a+bt) \quad t_1 \le t \le T \tag{2}$$

On solving equation (1) and (2) with boundary condition, we have

$$I(t) = e^{-\frac{\theta_0(\alpha)t^2}{2}}\left[a(t_1-t)+\frac{a\theta_0(\alpha)}{6}(t_1^3-t^3)+\frac{b\theta_0(\alpha)}{8}(t_1^4-t^4)\right] \quad 0 \le t \le t_1 \tag{3}$$

$$I(t) = a(t_1-t)+\frac{b}{2}(t_1^2-t^2) \quad t_1 \le t \le T \tag{4}$$

Using $I(0)=V$ in equation (3), we have

$$V = at + \frac{a\theta_0(\alpha)}{6}t_1^3 + \frac{b}{2}t_1^3 + \frac{b\theta_0(\alpha)}{8}t^4 \tag{5}$$

The total amount of deteriorated units in $0 \le t \le t_1$ is given by

$$\int_0^{t_1} \theta_0(\alpha)t\, I(t)\, dt$$

$$=\theta_0(\alpha)\left[\frac{V}{2}t_1^2 - \frac{\theta_0(\alpha)V}{8}t_1^4 - \frac{a}{3}t_1^3 - \frac{b}{8}t_1^4 + \frac{a\theta_0(\alpha)}{15}t_1^5 + \frac{b\theta_0(\alpha)}{48}t_1^6\right] \tag{6}$$

Holding cost per cycle is given by

$$C_h\int_0^{t_1} I(t)\, dt$$

$$= C_h\left[Vt_1 - \frac{a}{2}t_1^2 - \frac{b}{3}t_1^3 - \frac{\theta_0(\alpha)V}{6}t_1^3 + \frac{a\theta_0(\alpha)}{12}t_1^4 + \frac{b\theta_0(\alpha)}{40}t_1^5\right] \tag{7}$$

Deterioration cost per cycle is

$$C_d \int_0^{t} \theta_0(\alpha)\, I(t)\, dt$$

$$= C_d \theta_0(\alpha)\left[\frac{V}{2}t_1^2 - \frac{\theta_0(\alpha)V}{8}t_1^4 - \frac{a}{3}t_1^3 - \frac{b}{8}t_1^4 + \frac{a\theta_0(\alpha)}{15}t_1^5 + \frac{b\theta_0(\alpha)}{48}t_1^6\right] \quad (8)$$

Shortage cost per cycle is

$$C_s \int_{t_1}^{T} I(t)\, dt$$

$$= C_s\left[at_1T - \frac{a}{2}T^2 + \frac{b}{2}t_1^2T - \frac{b}{6}T^3 - \frac{a}{2}t_1^2 - \frac{b}{3}t_1^3\right] \quad (9)$$

Total average cost of the system is given by

$$TC(V, t_1, \alpha) = \frac{1}{T}\left[HC + DC - SC\right]$$

$$= \frac{1}{T}\left[C_h\left\{Vt_1 - \frac{a}{2}t_1^2 - \frac{b}{3}t_1^3 - \frac{\theta_0(\alpha)V}{6}t_1^3 + \frac{a\theta_0(\alpha)}{12}t_1^4 + \frac{b\theta_0(\alpha)}{40}t_1^5\right\}\right.$$

$$+\theta_0(\alpha)C_d\left\{\frac{V}{2}t_1^2 - \frac{\theta_0(\alpha)V}{8}t_1^4 - \frac{a}{3}t_1^3 - \frac{b}{8}t_1^4 + \frac{a\theta_0(\alpha)}{15}t_1^5 + \frac{b\theta_0(\alpha)}{48}t_1^6\right\}$$

$$\left.-C_s\left\{at_1T - \frac{a}{2}T^2 + \frac{b}{2}t_1^2T - \frac{b}{6}T^3 - \frac{a}{2}t_1^2 - \frac{b}{3}t_1^3\right\}\right] \quad (10)$$

Eliminating V from equation (10), we have

$$TC(t_1, \alpha) = \frac{1}{T}\left[\theta_0(\alpha)C_d\left\{\frac{1}{2}\left(at_1^3 + \frac{a\theta_0(\alpha)}{6}t_1^5 + \frac{b}{2}t_1^4 + \frac{b\theta_0(\alpha)}{8}t_1^6\right)\right.\right.$$

$$\left.-\frac{\theta_0(\alpha)}{8}\left\{at_1^5 + \frac{a\theta_0(\alpha)}{6}t_1^7 + \frac{b}{2}t_1^6 + \frac{b\theta_0(\alpha)}{8}t_1^8\right\} - \frac{a}{3}t_1^3 - \frac{b}{8}t_1^4 - \frac{a\theta_0(\alpha)}{15}t_1^5 + \frac{b\theta_0(\alpha)}{48}t_1^6\right\}$$

$$+C_h\left\{at_1^2+\frac{a\theta_0(\alpha)}{6}t_1^4+\frac{b}{2}t_1^3+\frac{b\theta_0(\alpha)}{8}t_1^5-\frac{\theta_0(\alpha)}{6}\left(at_1^4+\frac{a\theta_0(\alpha)}{6}t_1^6+\frac{b}{2}t_1^5+\frac{b\theta_0(\alpha)}{8}t_1^7\right)\right.$$

$$\left.-\frac{a}{2}t_1^2-\frac{b}{3}t_1^3+\frac{a\theta_0(\alpha)}{12}t_1^4+\frac{b\theta_0(\alpha)}{40}t_1^5\right\}-C_s\left\{at_1T-\frac{a}{2}T^2+\frac{b}{2}t_1^2T-\frac{b}{6}T^3-\frac{a}{2}t_1^2\right.$$

$$\left.\left.-\frac{b}{3}t_1^3\right\}\right] \tag{11}$$

Hence Mean cost $<C(t_1)>$ is obtained by the integral

$$<C(t_1)>=\int_\Gamma C(t_1,\alpha)\ p(\alpha)d\alpha$$

and substituting A for $\int_\Gamma \theta_0(\alpha)\ p(\alpha)d\alpha$

$$<C(t_1)>=\frac{1}{T}\left[\left[AC_d\left\{\frac{1}{2}\left(at_1^3+\frac{aA}{6}t_1^5+\frac{b}{2}t_1^4+\frac{bA}{8}t_1^6\right)-\frac{A}{8}\left(at_1^5+\frac{b}{2}t_1^6\right)\right\}-\frac{a}{3}t_1^3-\frac{b}{8}t_1^4\right.\right.$$

$$\left.\left.+\frac{bA}{40}t_1^5\right\}-C_s\left\{at_1T-\frac{a}{2}T^2+\frac{b}{2}t_1^2T-\frac{b}{6}T^3-\frac{a}{2}t_1^2-\frac{b}{3}t_1^3\right\}\right] \tag{12}$$

Neglecting higher power of $\theta_0(\alpha)$ i.e. two or more than two power of $\theta_0(\alpha)$.

For optimum minimum value of t_1, minimize equation (12)

Put $\frac{d}{dt}<C(t_1)>=0$

$$\left[\frac{AC_d}{2}\left\{3at_1^2+\frac{5aA}{6}t_1^4+2bt_1^3+\frac{3bA}{4}t_1^5\right\}-\frac{5Aa}{8}t_1^4-\frac{3bA}{8}t_1^5-at_1^2-\frac{b}{2}t_1^3-\frac{aA}{3}t_1^4+\frac{bA}{8}t_1^5\right]$$

$$+C_h\left\{2at_1+\frac{2aA}{3}t_1^3+\frac{3b}{2}t_1^2+\frac{5A}{8}t_1^4-\frac{2Aa}{3}t_1^3-\frac{5Ab}{12}t_1^4-at_1-bt_1^2+\frac{aA}{3}t_1^3+\frac{bA}{8}t_1^4\right\}$$

$$-C_s\left[aT+bTt_1-at_1-bt_1^2\right]=0 \tag{13}$$

Solving equation (13) by suitable method 0f theory of equations or by using Matlab software to determine optimal and hence the optimal mean cost by equation (12). Also initial inventory level can be calculated using and total amount of replenishment in the beginning of each cycle can be determined. *1*t* ><)(1*tC* **V* **Q*

4. CONCLUDING REMARKS:-

Here, we develop an inventory model for deteriorating items with linear demand, considering random rate of deterioration. The model is studied for minimization of total average cost.

5. REFERENCES:

(1) Covert, R.P. and Philip, G.C. (1973). An EOQ model for deteriorating items with weibull distributions deterioration. AIIE Transaction, vol. 5(4), 323-332.

(2) Deb, M. and Chaudhari, K.S. (1986). An EOQ model for items with finite rate of production and variable rate of deterioration. Opsearch, vol. 23(3), 175-181.

(3) Ghare, P.M. and Schrader, G.F. (1963). A model for an exponentially decaying inventory. Journal of Industrial Engineering, vol. 14, 238-243.

(4) Gupta, R. and Vart, P. (1986). Inventory model for stock dependent consumption rate. Opsearch, vol. 26(1), 19-24.

(5) Mandel, B.N. and Phaujdar, S. (1989). An inventory model for deteriorating items and stock-dependent consumption rate. J. Opl. Res. Soc., vol. 40(5), 483-488.

(6) Mandel, M. and De, S.K. (2012). An EOQ model with parabolic demand rate and time varying selling price. Analysis of Pure and Applied Mathematics, vol. 1, 32-43.

(7) Sharma, A.K., Aggarwal, N.K. and Khurana, S.K. (2015). An EOQ model for deteriorating items with ramp type demand, weibull distributed deterioration and shortages. Arybhatta Journal of Mathematics and Informatics , vol. 7(2), 297-304.

(8) Sharma, V. and Chaudhari, R.R. (2013). An inventory model for deteriorating items with weibull deterioration with time dependent demand and shortages. Research journal of Management Sciences, vol. 2(3), 28-30.

(9) Shikha, D., Kishan, H. and Rani, M. (2015). An inventory model of perishable products with displayed stock level dependent demand rate. Aryabhatta Journal of Mathematics and Informatics, vol. 7(2), 333-338.

(10) Singh, S. (2016). An inventory model for perishable items having constant demand with time dependent holding cost. Mathematics and Statistics, vol. 4(2), 58-61.

(11) Su, C.T., Tong, L.I. and Liao, H.C. (1996). An inventory model under inflation for stock dependent consumption rate and exponential decay. Opsearch, vol. 36(2), 71-82.