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A STUDY OF GENERALIZED TRANSFORMED DISTRIBUTION

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Abstract :

A generalized probability density function (p.d.f.) is obtained from the lograthemic transformation of a generalized hypergeometric p.d.f. Some interesting special cases, expression for m.g.f, moments about origin, distribution of ratio are also given. Some applications of this distribution are also pointed out.

Key Words : Probability density, lograthemic transformation, hypergeometric

1. THE DISTRIBUTION

In what follows the parameters and variables are restricted to take those values for which the density is non-negative.

The generalized lograthemic p.d.f. is given in the form as

$$f(y)=\begin{cases}\dfrac{y^{\beta-1}\left[\log\dfrac{1}{y}\right]^{\alpha-1}{}_pF_q\left(a_1,\dots,a_p;b_1,\dots,b_q;k\log\left(\dfrac{1}{y}\right)\right)}{\Gamma(\alpha)\beta^{-\alpha}{}_{p+1}F_q\left(a_1,\dots,a_p;\alpha;b_1,\dots,b_q;\dfrac{k}{\beta}\right)}, & \text{where } Re(\alpha)>0,\ Re(\beta)>0,\ Re(k)<Re(\beta) \\ & b_1,\ b_2\ \dots,\ b_q\neq 0,-1,-2,\dots,\ 0<y<1 \\ 0, \text{ elsewhere} \end{cases} \quad \dots(1.1)$$

2. SPECIAL CASES

(i) Putting $k = 0$, (1.1) takes the form

$$f(y)=\begin{cases}\dfrac{y^{\beta-1}\left[\log\left(\dfrac{1}{y}\right)\right]^{\alpha-1}}{\Gamma(\alpha)\,\beta^{-\alpha}}, & Re(\alpha)>0,\ Re(\beta)>0,\ 0<y<1 \\ 0, \text{ elsewhere}\end{cases} \quad \dots(2.1)$$

(ii) When $a_n = b_n$, $n \geq 3$, we get a p.d.f. due to Saxena and Mishra (6) as

$$f(y)=\begin{cases}\dfrac{y^{\beta-1}[\log\left(\dfrac{1}{y}\right)]^{\alpha-1}{}_2F_2\left(a_1,a_2;b_1,b_2;k\log\left(\dfrac{1}{y}\right)\right)}{\Gamma(\alpha)\beta^{-\alpha}{}_3F_2\left(a_1,a_2,\alpha;b_1,b_2;\dfrac{k}{\beta}\right)}, & \text{where } Re(\alpha)>0,\ Re(\beta)>0,\ Re(k)<Re(\beta) \\ & b_1,\ b_2\neq 0,-1,-2,\ \dots,\ 0<y<1 \\ 0, \text{ elsewhere}\end{cases} \quad \dots(2.2)$$

(iii)Putting $a_p = b_q$ in (1.1) we get

$$f(y)=\begin{cases}\dfrac{y^{\beta-1}\left[\log\left(\frac{1}{y}\right)\right]^{\alpha-1}\ {}_{p-1}F_{q-1}\left(a_1,a_2...,a_{p-1};b_1,b_2...b_{q-1};k\log(1/y)\right)}{\Gamma(\alpha)\beta^{-\alpha}\ {}_pF_{q-1}\left(a_1,a_2,...a_{p-1};\alpha;b_1,...,b_{q-1};k/\beta\right)}, & \text{where } Re(\alpha)>0,\ Re(\beta)>0,\ Re(k)<Re(\beta)\\ & b_1,b_2,...,b_{q-1}\neq 0,-1,-2....;\ 0<y<1\\ 0, \text{ elsewhere} & \end{cases} \quad ...(2.3)$$

(iv) By making $b_q = \alpha$, (1.1) reduces to

$$f(y)=\begin{cases}\dfrac{y^{\beta-1}\left[\log\left(\frac{1}{y}\right)\right]^{\alpha-1}\ {}_pF_q\left(a_1,...,a_p;b_1,b_2...,b_{q-1},\alpha;k\log(1/y)\right)}{\Gamma(\alpha)\beta^{-\alpha}\ {}_pF_{q-1}\left(a_1,...a_p;b_1...,b_{q-1};k/\beta\right)}, & \text{where } Re(\alpha)>0,\ Re(\beta)>0,\ Re(k)<Re(\beta)\\ & b_1,b_2.....b_{q-1}\neq 0,-1,-2,.....;\ 0<y<1\\ 0, \text{ elsewhere} & \end{cases} \quad ...(2.4)$$

3. THE r^{th} ORDER MOMENT ABOUT ORIGIN

The r^{th} order moment about origin of the random variable (r.v.) y following the p.d.f.(1.1) is given by

$$\mu'_r = E(y^r) = \int_0^1 y^r f(y)dy$$

$$= \frac{\beta^\alpha}{(\beta+r)^\alpha}\frac{{}_{p+1}F_q(a_1,a_2,...,a_p,\alpha;b_1,...;b_q;k/\beta+r)}{{}_{p+1}F_q(a_1,...,a_p,\alpha;b_1,...;b_q;k/\beta)}$$

where $Re(\alpha)>0$, $Re(\beta)>0$, $Re(k)<Re(\beta)$

$b_1, b_2 .... b_q \neq 0, -1, -2, ...;\ r = 0, 1, 2, ...$ ...(3.1)

4. THE MOMENT GENERATING FUNCTION

The m.g.f. of the r.v. y following the p.d.f. (1.1) is given by

$$M_y(t) = E(e^{ty}) = \int_0^1 e^{ty} f(y)dy$$

$$=\beta^\alpha\sum_{j=0}^{\infty}\frac{t^j}{j!}\frac{{}_{p+1}F_q(a_1,...,a_p;\alpha;b_1,...,b_q;k/\beta+j)}{(\beta+j)^\alpha\ {}_{p+1}F_q(a_1,...,a_p,\alpha;b_1,....,b_q;k/\beta)}$$

where $Re(\alpha)>0$, $Re(\beta)>0$, $Re(k)<Re(\beta)$

$b_1, b_2 ...., b_q \neq 0, -1, -2,.....$ ...(4.1)

5. A GENERALIZED COMPOUND TYPE I GEOMETRIC DISTRIBUTION

Let χ be a r.v. which follows a type I geometric distribution conditional on t, i.e;

$$P\left(\chi=\frac{x}{t}\right)=\begin{cases}t(1-t)^{x-1}, x=1,2,3,.....;0<t<1\\ 0, \text{ elsewhere}\end{cases} \quad ...(5.1)$$

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It t follows a p.d.f. of the form (1.1), then the unconditional probability function of χ is given by

$$P(\chi = x) = \begin{cases} \dfrac{\beta^{\alpha}}{p+1^{F}q\left(a_1,\ldots,a_p,\alpha;b_1,\ldots b_q;k/\beta\right)} \\ \cdot\sum_{j=0}^{\infty}\dfrac{(-1)^j p+1^F q\left(a_1,\ldots,a_p,\alpha;b_1,\ldots,b_q;k/(\beta+1+j)\right)}{(\beta+1+j)^{\alpha}}\binom{x-1}{j} \\ \text{Where } Re(\alpha) > 0,\ Re(\beta) > 0,\ Re(K) < Re(\beta) \\ b_1,\ b_2,\ \ldots\ b_q \neq 0, -1, -2, \ldots;\ x = 1, 2, 3 \\ 0, \text{ elsewhere} \end{cases} \quad \ldots(5.2)$$

several special cases of (5.2) can be worked out for specific values of the parameter e.g., when k = 0, (5.2) reduces to,

$$P(\chi = x) = \begin{cases} \beta^{\alpha}\sum_{j=0}^{x-1}\dfrac{(-1)^j\binom{x-1}{j}}{(\beta+1+j)^{\alpha}}, \text{ where } Re(\alpha) > 0,\ Re(\beta) > 0;\ x = 1, 2, 3 \\ 0, \text{ elsewhere} \end{cases} \quad \ldots(5.3)$$

The form (5.3) has been used to describe the probablistic nature of waiting time in fertility analysis as an alternative to betageometric distribution, given by saxena and mishra (6), exactly on the same lines as given by mishra (4).

6. THE DISTRIBUTION OF RATIO

We will consider the distribution of ratio of two independent stochastic variables having the probability density functions belonging to the same family as f(x). Let x_1 and x_2 be two independent stochastic variables with probability functions -

$$f(x_i) = \begin{cases} \dfrac{e^{-\beta_i x_i} x_i^{\alpha_i - 1} p^F q\left(a_1,\ldots,a_p;b_1,\ldots,b_q;kx_i\right)}{\Gamma(\alpha_i)\beta_i^{-\alpha_i} p+1^F q\left(a_1,\ldots,a_p,\alpha_i;b_1,\ldots,b_q;k(\beta_i)\right)}, \text{ where } Re(\alpha_i) > 0,\ Re(\beta_i) > 0,\ Re(k) < Re(\beta_i) \\ \qquad b_1,\ b_2, \ldots, b_q \neq 0, -1, \ldots;\ 0 < x < \infty \\ 0, \text{ elsewhere} \end{cases} \quad \ldots(6.1)$$

Let $W = x_1 / x_2$, then $V = \log W = \log x_1 - \log x_2$

The characterstic function of V is

$\phi_v(t) = E\left(e^{wtv}\right)$, $w = F_1$

$$= \int_0^{\infty}\int_0^{\infty} e^{wt(\log x_1 - \log x_2)} f(x_1) f(x_2)\, dx_1\, dx_2$$

$$= \frac{G\left(\alpha_1 + wt, \beta_1, a_1, \ldots, a_p, b_1, \ldots, b_q, k/\beta_1\right).G\left(\alpha_2 - wt, \beta_2, a_1, \ldots; a_p, b_1 \ldots b_q, k/\beta_2\right)}{\prod_{j=1}^{2} G\left(\alpha_j, \beta_j, a_1, \ldots, a_p, b_1, \ldots, b_q; k/\beta_j\right)} \quad \ldots(6.2)$$

where $G(\alpha_j, \beta_j, a_1,...,a_p, b_1....b_q, k/\beta_j)$

$$= \Gamma(\alpha_j)\beta_j^{-\alpha_j} {}_pF_q\left(\alpha_j, a_1,...,a_p; b_1,..,b_q; k/\beta_j\right) \quad ...(6.3)$$

The fourier transform of $\phi_v(t)$ gives the density function of V and by making the transformation $W = e^v$, we obtain the density function of W, which is given by

$$\phi_w(t) = \begin{cases} \displaystyle\int_{-\infty}^{\infty} \frac{W^{-wx} G\left(\alpha_j + wt, a_1,...a_p, b_1,...b_q, k/\beta_1\right) G\left(\alpha_2 - wt, a_1,...,a_p, b_1,...,b_q, k/\beta_2\right)}{\prod_{j=1}^{2} G\left(\alpha_j, a_1,...a_p, b_1....b_q, k/\beta_j\right)} dx \\ \text{when } Re(\alpha_i) > 0,\ Re(\beta_i) > 0,\ Re(k) < Re(\beta_j) \\ b_1, b_2,....b_q \neq 0, -1, ...;\ 0 < x < \infty \\ 0, \text{ elsewhere} \end{cases} \quad ...(6.4)$$

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