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An EOQ model for non-instantaneous items with quadratic time dependent demand, Weibull type deterioration and linear holding cost under trade credits with shortages

Ravikant Sharma
Department of Mathematics, Raj Rishi Govt. College, Alwar– 301001 (Rajasthan), India
drravikant15370@gmail.com

Abstract: In today's business environment, a supplier usually offers Permissible delay in payments mainly to promote market competition and the holding cost changes as on-hand inventory changes with time. This study investigates an EOQ inventory model for non-spontaneous Weibull deteriorating commodities with quadratic time-induced demand rate and linearly time-sensitive carrying cost under trade credits. Shortages are permitted and partially backlogged, where the backlogging rate is a function of the hold-up time for the upcoming order. Mathematical model is demonstrated to finding optimal replenishment policy to obtain maximum total profit. The model is discussed under three conditions that depends on the duration of non-deterioration, the time at which shortages take place, permissible delay period in payments and replenishment cycle length. This study mainly shows the impact of order quantity and permissible delay period on total profit. The outcomes are demonstrated by sensitivity analysis of parameters and exhibited it with graphs and also exemplify with numerical examples for each case.

Keywords: non-spontaneous Weibull deteriorating commodities, quadratic time-induced demand rate, linearly time-sensitive carrying cost, trade credits, Shortages, partially backlogged.

1. Introduction

Inventory policy plays a major role in any industry. Hence, set up of a proper inventory policy counting various aspect is essential. Types of demand and Deterioration, Shortages, delay in payments, inflation etc. are some of those aspects. Some commodities decays spontaneous whereas many products such as packaged foods and dry fruits, chemicals, pharmaceuticals, perfumes etc. starts deteriorating after a fix time of their packaging. So, study of spontaneous and non-spontaneous type of deterioration is equally important. In day-to-day life, price of goods itself, prices of other related goods, population, climate and weather, income distribution etc. are the factors that changes the demand of a commodity with time, which is unavoidable. In an inventory structure costs of financing, damage, obsolescence, warehousing, handling inventory affects holding cost and because of demand and decay it variate with time. So, it is quite wise to take a variable carrying cost. To promote market competition and sometimes as a substitute of price discount, suppliers provides credit period to the retailer. A number of researchers have done significant work in this direction including the aforementioned aspects. For non-instantaneous deteriorating goods, [1] established a multi-item optimal joint replenishment model under constant demand rate allowing full backlogging. Inventory systems were framed by [16] and [20] by assuming defective quality of products and a cash discount respectively. [21] presented the optimal ordering policies under trade credit policy and [6] provided consummate proof for [21]'s inventory model. By taking expiry date and selling price-based demand and linearly time linked carrying cost, [28] investigated an inventory framework for commodities which deteriorates and expires with time considering both constant and changing partial backlogging rate. Electronic products deteriorates due to technology development and sometimes due to breakage. With ramp-kind of demand, Weibull deterioration and shortages, [30] presented an EOQ model which is also capable for seasonal and latest introduced high-tech products Whereas [38] obtained optimal replenishment policy for a single product and [3] investigated a two-storage inventory framework with trade credits under rise of prices.

Considering stock based demand and variable carrying cost, [36] obtained optimal ordering policy and discussed it with and without shortages as special cases whereas [24] presented a two-echelon inventory model assuming that during the shortages, retailer provides a price discount to consumers who cancels their orders and the supplier and retailer both allows delay in payments to their customers to promote the market competition. With price linked demand and

Weibull allocation decline, [35] investigated an inventory pattern by taking time linked carrying cost, with and without shortages and shortages are completely backlogged whenever they are allowed whereas with same type of demand and decline under trade credit policy, [34] suggested an EOQ model and [12] formulated a fuzzy inventory for non-spontaneous decaying products. [12] Also used the fact that the supplier provides relaxation in balancing the account at the finishing of credit duration and also considered the chances of high rate of earned interest compare to interest payable. [33] presented different types of sales. By Assuming price and time linked demand [9] established an economic production quantity model dealing with non-spontaneous deterioration nature of commodities with inflation under selling price and demand induced customer returns and also studied the impact of TVM based on Discounted Cash Flow approach. With similar type of demand and time linked decay rate under partial backlogging [8] framed a model that deals with pricing and inventory limitation and [32] studied effects of credit period, shortages and inflation on an EOQ model under partially backlogged shortages.

In this era of advertisement, its cost and frequency seriously affects an inventory model. With recurrence of advertisement and price dependent demand, general kind of decline and carrying cost [26] presented a marketing policy included inventory model and with advertisement cost and price based demand [23] developed an EOQ model with credit period subject to inflation and TVM. With same type of demand, [37] framed an inventory model considering credit period, time discounting and salvage value of deteriorated units under inflation.

In a different case, [11] discussed inventory framework for two decaying substitute products where demand for a substitute commodity is proportional to other's price and own stock and inversely proportional to its own price and other's stock. With promotional efforts and price dependent demand [7] developed a cost minimization model. By assuming selling price and inflation dependent demand rate, [10] established an inventory model for non-instantaneous perishable products with trade credits, inflation, partially backlogged shortages and customer returns proportional to the sold quantity and product price. Two-warehouse inventory models for non-spontaneous perishable products were proposed by [29] (with stock linked demand) and [27] (by assuming stock and interval-valued inventory prices induced demand rate and shortages with inflation). By taking non-linear stock linked demand [2] derived an EOQ model with non-linear stock sensitive carrying cost and permissible delay in payments under partially backlogged shortages and [25] presented a stock-review framework with stock sensitive demand, consistent restock rate and quadratic time based decline. In a similar way, [41] discussed an inventory model for non-spontaneous perishable products with stock linked demand and partially backlogged shortages. [5] modified [41] by firstly, setting the purpose to optimal total profit, secondly, to counter the fact that most retail stores have restricted self-space, they set a maximum inventory level, thirdly, when shortages are not profitable, there is no provision of zero ending inventory. further, [31] extended [5] by taking selling price and inventory level induced demand with trade credit policy. [40] complemented [31] by selling ending inventory as salvages with assuming all possibilities of replenishment cycle length including shorter than the duration of non-decay.

With time induced demand and carrying cost, inventory models were proposed by [18] for those business units that utilize the conservation technology to decrease the deterioration rate of the spontaneous deteriorate products and by [19] for the business organizations where carrying cost and decline rate both are time dependent with time proportional deterioration and partially backlogged shortages. further, with linearly time linked carrying cost and Weibull deterioration, [39] framed inventory models dealing with production by assuming quadratic time based demand with Salvage value and without shortages whereas [22] presented an inventory pattern for non-spontaneous decaying products by assuming power pattern type of demand under partially backlogged shortages and trade credit policy.

Considering the fact that to boost the market competition a retailer who gets permissible delay in payments provided by his/her supplier also provides this to his/her consumers, [15] studied the effects of delay in payments on the classic EPQ model for an item with exponential demand whereas [13] analysed this impact for Weibull decaying products in two-storage environment with partially backlogged shortage. Similarly, [4] investigated the effects of the backlogging rate on the EOQ decisions subject to partial backlogged time linked demand. [17] studied an inventory framework with partially backlogged time-induced demand for non-spontaneous decaying products and they concluded that replenishment cycle time is unique and free from type of demand and [14] derived an inventory pattern for a product with quadratic time based demand and shortages where delay in payments are allowed.

Study by	Deterioration	Type of Demand rate	Type of Deterioration	Holding cost	Delay in Payments	Shortages	Objective
Farughi	Non-	Price	Time	Constant	No	Partial	Profit
et. al (2014)	instantaneous	and time	dependent			backlogged	
Jaggi	Non-	price	Constant	Constant	Yes	No	Profit
et. al (2015)	instantaneous						
Chakraborty	instantaneous	Ramp - type	Weibull type	Constant	Yes	Partial	Cost
et. al (2018)			(3 parameter)			backlogged	
Venkateswarlu	instantaneous	Time (quadratic)	Weibull type	Linearly time-	No	No	Profit
& Reddy (2016)			(2 parameter)	dependent			
Valliathal &	Non-	Ramp-type	Weibull type	Constant	No	Partial	Cost
Uthayakumar	instantaneous					backlogged	
(2016)							
Tripathy &	instantaneous	Price	Weibull type	Linearly time-	No	With & without	Profit
Mishra (2010)			(2 parameter)	dependent		shortage (fully	
						backlogged)	
Tripathi &	Non-	Price	Weibull type	Constant	Yes	No	Profit
Pandey (2020)	instantaneous						
Singh	instantaneous	Ramp-type	Weibull type	Constant		Fully	Cost
et. al (2018)			(3 parameter)			backlogged	
Shaikh	Non-	Interval valued	Constant	Constant	No	Partial	Cost
et. al (2019)	instantaneous	inventory				backlogged	
cost & stock							
Udaykumar	Non-	Selling price &	Constant	Constant	yes	Partial	Cost
et. al (2020)	instantaneous	advertisement cost				backlogged	
Sundararajan	instantaneous	Time and price	Constant	Constant	Yes	With & without	Profit
et. al (2020)						shortages	
Present study	Non-	Time (quadratic)	Weibull type	Linearly time-	Yes	Partial	Profit
	instantaneous		(2 parameter)	dependent		backlogged	

1.1 Problem Narration

The primary purpose of this study is to investigate an optimal replenishment strategy for non-spontaneous deteriorating products over infinite time horizon to obtain optimal total profit. For this, in the present work, we developed an EOQ model with quadratic time dependent demand, two-parameter Weibull type decline rate where shortages are allowed and partial backlogged with trade credit policy and carrying cost linearly depends on time.

The remaining part of the aforementioned study is organised in this manner: section 2 contains postulations and symbols implemented in formulation of this model. Mathematical model and solutions of this model depicted in section 3. Solution procedure and Numerical examples is presented in section 4 and 5 respectively. Sensitivity analysis of decision variables with respect to the parameters of this paper is shown in section 6 and observations based on this analysis is discussed in section 7. With suggestions for potential future research this paper is concluded in section 8.

2. Modeling Assumption and Notations

The underneath postulations and notations are assumed to develop the mathematical model of this model

1. This inventory systems has particular non-direct failing item.
2. The lead time is negligible and replenishment take place instantaneously at an infinite rate.
3. The demand D is a quadratic function of time 't', that is $D(t) = at^2 + bt + c$.
4. Holding cost has taken as a linearly dependent on time,that is $h = d + et$, $d > 0$ and $e > 0$.
5. Backorder starts after time t_2.
6. Shortages are tolerated with partially backlogging rate for negative inventory is unsteady and depends on the length of the hold back time for the upcoming restock and it is symbolized by $B(t) = \frac{1}{e^{\delta(T-t)}}$, where $\delta > 0$ and $t_2 \leq t \leq T$.
7. It is supposed that during period $[0, t_1]$,the commodities has no deterioration. After this period the on-hand inventory level deteriorates with two parameter Weibull decay $\theta = \alpha\beta t^{1-\beta}$, $\alpha \geq 0, \beta \geq 0$.
8. Repairing or replacing for the deteriorated units is not allowed.
9. During the trade credit duration, the retailer need not have to settle down the account with the supplier, the retailer saves earned sales revenue in an interest generating account. The supplier starts charging interest after the trade credit period M.

k	ordering cost per order
R	The retailer's Order quantity
$D(t)$	demand at time t
t_1	Time at which commodities start to deteriorate
t_2	Time to exhaust stock within a replenishment cycle

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T	Replenishment cycle time
M	the Permissible delay period
p	Unit purchasing cost of an item
s	Selling price
h	Holding cost
c_2	Shortage cost per unit per cycle
c_0	Opportunity cost of lost sales
I_p	The interest charged per unit of money
I_e	The interest earned per unit of money
$\theta(t)$	The rate of deterioration
P	The maximum amount of demand backlogged
δ	The backlogging parameter which is a positive fixed value
$I_1(t)$	Inventory level in the time interval $(0 \le t \le t_1)$
$I_2(t)$	Inventory level in the time interval $(t_1 \le t \le t_2)$
$I_3(t)$	Inventory level in the time interval $(t_2 \le t \le T)$

3. Mathematical Formulation and Solution of the Model

In this segment, the mathematical formulation and solutions of this model are discussed. At the beginning of the cycle retailer orders and receives R quantity of goods. In time interval $[0, t_1]$, the inventory level denoted by $I_1(t)$ and decline only due to demand. After that the inventory level declines because of demand and the decay in interval $[t_1, t_2]$ and it is referred by $I_2(t)$. At $t = t_2$, inventory pass on zero and after that shortages happen in time interval $[t_2, T]$ and in this interval is denoted by $I_3(t)$.

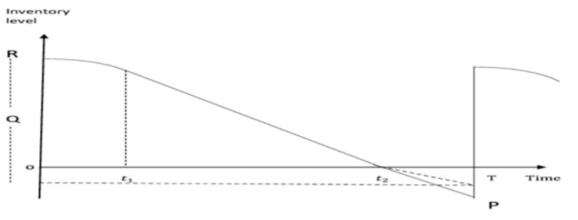

Figure 1: Graphical portrayal of the inventory framework (see online version for colours)

Therefore, the change in inventory level with time throughout the time interval $[0, t_2]$ is governed by the underneath differential equations:

$$\frac{dI_1(t)}{dt} = -(at^2 + bt + c); 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} + \alpha\beta t^{\beta-1} I_2(t) = -(at^2 + bt + c); t_1 \le t \le t_2 \tag{2}$$

With the boundary conditions $I_1(0) = R$, $I_2(t_2) = 0$, the solutions of equations (1) and (2) are

$$I_1(t) = R - \left(\frac{a}{3}t^3 + \frac{b}{2}t^2 + ct\right) \tag{3}$$

and

$$I_2(t) = x_1 + \alpha x_2 - \alpha x_1 t^\beta - \left(\frac{a}{3}t^3 + \frac{b}{2}t^2 + ct\right) + \alpha\beta\left(\frac{a}{\beta+3}t^{\beta+3} + \frac{b}{\beta+2}t^{\beta+2} + \frac{c}{\beta+1}t^{\beta+1}\right) \tag{4}$$

where

$$x_1 = \frac{a}{3}t_2^3 + \frac{b}{2}t_2^2 + ct_2$$

$$x_2 = \frac{a}{\beta+3}t_2^{\beta+3} + \frac{b}{\beta+2}t_2^{\beta+2} + \frac{c}{\beta+1}t_2^{\beta+1} \tag{5}$$

Here I(t) is a continuous function of 't'. Therefore at $t = t_1$, we have $I_1(t_1) = I_2(t_1)$. With the help of this relation, from equations (3) and (4), we find

$$R = x_1 + \alpha x_2 - \alpha x_1 t_1^\beta + \alpha\beta\left(\frac{a}{\beta+3}t_1^{\beta+3} + \frac{b}{\beta+2}t_1^{\beta+2} + \frac{c}{\beta+1}t_1^{\beta+1}\right) \tag{6}$$

Partial Backlogging Model

The inventory level $I_3(t)$ during the shortage period $[t_2, T]$ is ruled by the upcoming differential equation

$$\frac{dI_3(t)}{dt} = -\frac{at^2+bt+c}{1+\delta(T-t)}; t_2 \le t \le T \tag{7}$$

On applying the condition $I_3(t_2) = 0$, the solution of equation (7) is

$$I_3(t) = e^{-\delta(T-t_2)}\frac{[\delta^2(at_2^2 + bt_2 + c) - \delta(2at_2 + b) + 2a]}{\delta^3}$$
$$-e^{-\delta(T-t)}\frac{[\delta^2(at^2+bt+c)-\delta(2at+b)+2a]}{\delta^3} \tag{8}$$

Using boundary condition $I_3(T) = -P$, we get the negative inventory

$$P = \frac{[\delta^2(aT^2 + bT + c) - \delta(2aT + b) + 2a]}{\delta^3}$$
$$-e^{-\delta(T-t_2)}\frac{[\delta^2(at_2^2+bt_2+c)-\delta(2at_2+b)+2a]}{\delta^3} \tag{9}$$

Total inventory, $Q = R + P$

$$Q = x_1 + \alpha x_2 - \alpha x_1 t_1^\beta + \alpha\beta\left(\frac{a}{\beta+3}t_1^{\beta+3} + \frac{b}{\beta+2}t_1^{\beta+2} + \frac{c}{\beta+1}t_1^{\beta+1}\right)$$
$$\frac{+[\delta^2(aT^2 + bT + c) - \delta(2aT + b) + 2a]}{\delta^3}$$
$$-e^{-\delta(T-t_2)}\frac{[\delta^2(at_2^2+bt_2+c)-\delta(2at_2+b)+2a]}{\delta^3} \tag{10}$$

The components of the total profit (TP) per cycle are :

(i) Ordering Cost:

$$OC = k$$

(ii) Purchase Cost

$$PC = pR$$
$$p\left[x_1 + \alpha x_2 - \alpha x_1 t_1^\beta + \alpha\beta\left(\frac{a}{\beta+3}t_1^{\beta+3} + \frac{b}{\beta+2}t_1^{\beta+2} + \frac{c}{\beta+1}t_1^{\beta+1}\right)\right] \tag{11}$$

(iii) Holding Cost:

$$HC = \int_0^{t_1}(d + et)\, I_1(t)dt + \int_{t_1}^{t_2}(d + et)\, I_2(t)dt$$
$$(x_1 + \alpha x_2)\left(dt_2 + \frac{et_1^2}{2}\right) + \frac{\alpha\beta}{(\beta+5)}\left[\frac{ae}{(\beta+3)}\right]\left(t_2^{\beta+5} - t_1^{\beta+5}\right)$$
$$\frac{+\alpha\beta}{(\beta+4)}\left[\frac{ad}{(\beta+3)} + \frac{be}{(\beta+2)}\right]\left(t_2^{\beta+4} - t_1^{\beta+4}\right)$$
$$\frac{+\alpha\beta}{(\beta+3)}\left[\frac{bd}{(\beta+2)} + \frac{ce}{(\beta+1)}\right]\left(t_2^{\beta+3} - t_1^{\beta+3}\right)$$
$$\frac{+\alpha}{(\beta+2)}\left[\frac{cd\beta}{(\beta+1)} - ex_1\right]\left(t_2^{\beta+2} - t_1^{\beta+2}\right)$$
$$-\frac{d\alpha x_1}{(\beta+1)}\left(t_2^{\beta+1} - t_1^{\beta+1}\right)$$
$$-\frac{t_2^5}{15}ae - \frac{t_2^4}{24}(2ad + 3be) - \frac{t_2^3}{6}(bd + 2ce) - \frac{t_1^2}{2}cd$$
$$\frac{+t_1}{2}(2d + et_1)\left[\alpha\beta\left(\frac{a}{\beta+3}t_1^{\beta+3} + \frac{b}{\beta+2}t_1^{\beta+2} + \frac{c}{\beta+1}t_1^{\beta+1}\right) - \alpha x_1 t_1^\beta\right] \tag{12}$$

(iv) Sales Revenue:

$$SR = s\int_0^{t_2} D\,(t)dt + s\int_{t_2}^{T} D\,(t)B(T-t)dt$$
$$s\{x_1 + \frac{a[(T^2\delta^2 - 2T\delta + 2) - e^{-\delta(T-t_2)}(t_2^2\delta^2 - 2t_2\delta + 2)]}{\delta^3}$$
$$\frac{+b[(T\delta - 1) - e^{-\delta(T-t_2)}(t_2\delta - 1)]}{\delta^2}$$
$$-\frac{c(e^{-\delta(T-t_2)}-1)}{\delta}\} \tag{13}$$

(v) Shortage Cost:

$$SC = c_2\int_{t_2}^{T}[-I_3(t)]\,dt$$
$$-c_2\{\frac{a[-(T^2\delta^2 - 4T\delta + 6) + e^{-\delta(T-t_2)}(t_2^2\delta^2 - 4t_2\delta + 6)]}{\delta^4}$$
$$-\frac{b[(T\delta - 2) - e^{-\delta(T-t_2)}(t_2\delta - 2)]}{\delta^3}$$
$$\frac{+(T - t_2)e^{-\delta(T-t_2)}[\delta^2(at_2^2 + bt_2 + c) - \delta(2at_2 + b) + 2a]}{\delta^3}$$
$$\frac{+c[e^{-\delta(T-t_2)}-1]}{\delta^2}\} \tag{14}$$

(vi) Cost of Lost Sales:

$$CLS = c_0\int_{t_2}^{T} D\,(t)[1 - e^{-\delta(T-t)}]dt$$
$$c_0\{\frac{aT^3}{3} + \frac{bT^2}{2} + cT - x_1$$
$$-\frac{a[(T^2\delta^2 - 2T\delta + 2) - e^{-\delta(T-t_2)}(t_2^2\delta^2 - 2t_2\delta + 2)]}{\delta^3}$$
$$-\frac{b[(T\delta - 1) - e^{-\delta(T-t_2)}(t_2\delta - 1)]}{\delta^2}$$
$$-\frac{c[1-e^{-\delta(T-t_2)}]}{\delta}\} \tag{15}$$

According to our assumptions, there are these three possible cases:

Case 1:

In present occurrence we speculate that the trade credit policy period M is less or equal than the duration of non-deterioration i.e, $(0 < M \le t_1)$. In the aforementioned case

Interest payable

$$IP_1 = pI_p\left[\int_M^{t_1} I_1\,(t)dt + \int_{t_1}^{t_2} I_2\,(t)dt\right]$$
$$pI_p\langle\alpha\beta$$
$$\frac{+c}{(\beta+2)(\beta+1)}\left(t_2^{\beta+2} - t_1^{\beta+2}\right) - \frac{\alpha x_1}{(\beta+1)}\left(t_2^{\beta+1} - t_1^{\beta+1}\right)$$
$$+(x_1 + \alpha x_2)(t_2 - M)$$
$$+(t_1 - M)\left[\alpha\beta\left(\frac{a}{\beta+3}t_1^{\beta+3} + \frac{b}{\beta+2}t_1^{\beta+2} + \frac{c}{\beta+1}t_1^{\beta+1}\right) - \alpha x_1 t_1^\beta\right]$$
$$\frac{+M^2}{2}\left(\frac{a}{2}M^2 + \frac{b}{3}M + c\right) - \frac{t_2^2}{2}\left(\frac{a}{2}t_2^2 + \frac{b}{3}t_2 + c\right)\rangle \tag{16}$$

Interest earned

$$IE_1 = sI_e \int_0^M t\,D(t)dt$$

$\frac{sI_eM^2}{12}(3aM^3 + 4bM + 6c)$ (17)

Now, total profit per unit time per cycle (TP_1) is

$\frac{1}{T}[(SR + IE_1) - (OC + PC + HC + SC + CLS + IP_1)]$ (18)

Case 2:

In present assumption, the credit period M is greater than t_1 and less or equal than t_2, the time when inventory level vanishes i. e $(t_1 < M \leq t_2)$. For this case

Interest payable

$$IP_2 = pI_p \int_M^{t_2} I_2(t)dt$$

$$pI_p\{\frac{a\alpha\beta}{(\beta+4)(\beta+3)}\left(t_2^{\beta+4} - M^{\beta+4}\right) + \frac{b\alpha\beta}{(\beta+3)(\beta+2)}\left(t_2^{\beta+3} - M^{\beta+3}\right)$$
$$\frac{+c\alpha\beta}{(\beta+2)(\beta+1)}\left(t_2^{\beta+2} - M^{\beta+2}\right) - \frac{\alpha x_1}{(\beta+1)}\left(t_2^{\beta+1} - M^{\beta+1}\right)$$
$$+(x_1 + \alpha x_2)(t_2 - M)$$

$\frac{+M^2}{2}\left(\frac{a}{2}M^2 + \frac{b}{3}M + c\right) - \frac{t_2^2}{2}\left(\frac{a}{2}t_2^2 + \frac{b}{3}t_2 + c\right)\}$ (19)

Interest earned

$$IE_2 = sI_e \int_0^M t\,D(t)dt$$

$\frac{sM^2I_e}{12}(3aM^3 + 4bM + 6c)$ (20)

Now, for this case total profit per unit time per cycle (TP_2) is

$\frac{1}{T}[(SR + IE_2) - (OC + PC + HC + SC + CLS + IP_2)]$ (21)

Case 3:

In present situation, the credit period M exists between t_2 and cycle length T i.e $(t_2 < M < T)$. For the aforementioned case

Interest payable

$IP_3 = 0$ (22)

Interest earned

$$IE_3 = sI_e\left[\int_0^{t_2} t\,D(t)dt + (M - t_2)\int_0^{t_2} D(t)dt\right]$$

$sI_e\left[\frac{t_2^2}{12}(3at_2^3 + 4bt_2 + 6c) + x_1(M - t_2)\right]$ (23)

Now, for this case total profit per unit time per cycle (TP_3) is

$\frac{1}{T}[(SR + IE_3) - (OC + PC + HC + SC + CLS)]$ (24)

4. Solution Procedure

Step1: First of all partially differentiate equations (18), (21) and (24) with respect to t_t, t_2 and T.

Step2: Set the derivatives equal to zero.

Step3: Find possible values of t_1, t_2 and T.

Step4: Find second order partial derivatives of equations (18), (21) and (24) with respect to t_t, t_2 and T.

Step5: Check the signs of second order derivatives of step 4 at the values of t_t, t_2 and T found in step 3.

Step6: If the signs in step 5 are negative then TP_1, TP_2 and TP_3 will be maximum and these particular values of t_t, t_2, T will be optimal values.

By solving above derivative equations with the assistance of MATLAB software, the values of t_1, t_2, T^* and then from Equations. (10), (18), (21) and (24), the optimal values of Q^* and TP_1, TP_2 and TP_3 can be found out.

5. Numerical Examples

Example 1: In view of case 1 $(0 < M \leq t_1)$, let us assume the values of parameters like this: $a = 45, b = 150, c = 2, d = 13.4, e = 0.75, \alpha = 1.5, \beta = 0.56, \delta = 0.0001, k = 150\$, p = \$25/unit, s = \$73.4228/unit, c_2 = \$0.005/unit, c_0 = \$26/unit, I_p = 0.020/\$/year, I_e = 0.21/\$/year$ and $M = 0.004 years$.

Using MATLAB software for equations (10) optimum solution are

Optimal time $t_1 = 0.1283 years$, $t_2 = 0.1422 years$, $T^* = 1.9071 years$.

Maximum total profit$(TP_1) = \$14547/order/year$.

Economic order quantity$(Q^*) = 380.7440 units$.

Example 2: In view of case 2 $(t_1 < M \leq t_2)$, The assumptions are identical as first example except $a = 56, M = 0.02 years$. After solving, we get the following optimal results: $t_1 = 0.0058 years$, $t_2 = 0.3431 years$, $T^* = 1.8361 years$, $TP_2 = \$14555/order/year$, $Q^* = 377.7642 units$.

Example 3: In view of case 3 $(M > t_2)$, The assumptions are identical as example 1 except $a = 181, M = 0.80 years$. After solving, we get the following optimal values: $t_1 = 0.0050 years$, $t_2 = 0.3604 years$, $T^* = 1.8713 years$, $TP_3 = \$25644/order/year$, $Q^* = 669.4748 units$.

6. Sensitivity Analysis

In present part of the paper, sensitivity analysis is exhibited for all the parameters $a, b, c, d, e, \alpha, \beta, \delta, p, k, c_2, c_0, I_p, I_e$ and M used in this inventory system to find out the influence that changes in those parameters have on the expected total profit per unit time and the optimal values of t_1, t_2, T and Q. The sensitivity analysis is carried out by variate each parameter by +50%, +25%, -25%, and -50%, assuming single parameter as a variable at a instant and holding other parameters constant. Outcomes of this analysis is summarized in table 1, 2 and 3.

Table 2: Sensitivity analysis for the numerous parameters of this inventory model Case 1 $(0 < M \leq t_1)$

Parameters	Change in parameters	t_1	t_2	T	Total Profit (TP_1)	Q
a	50%	0.0202	0.3471	1.8454	12501	326.7381

	-25%	0.1283	0.1422	1.9071	14547	380.7440
	+25%	0.0200	0.7400	1.6630	12503	369.5423
	+50%	0.0200	0.7400	1.6630	13447	394.3900
b	-50%	0.0201	0.3560	1.8631	8904.80	234.4512
	-25%	0.0200	0.7399	1.6630	8700.50	260.4536
	+25%	0.0200	0.7399	1.6630	12606	381.5578
	+50%	0.0201	0.0386	2.1357	22815	665.8081
c	-50%	0.0200	0.7399	1.6630	10564	317.6804
	-25%	0.0200	0.7399	1.6630	10589	318.6995
	+25%	0.0201	0.3481	1.8473	13790	359.9325
	+50%	0.0496	0.3726	1.8500	13827	361.7884
d	-50%	0.0494	0.3479	1.8643	13956	365.4175
	-25%	0.0207	0.3426	1.8352	13660	353.6698
	+25%	0.0201	0.3471	1.8452	13739	358.1842
	+50%	0.0200	0.7399	1.6630	10471	319.9119
e	-50%	0.0200	0.7399	1.6630	10620	319.9119
	-25%	0.0200	0.7399	1.6630	10619	319.9119
	+25%	0.0218	0.3421	1.8339	13643	353.0779
	+50%	0.0208	0.1268	1.8533	14028	357.0464
α	-50%	0.0200	0.7400	1.6631	10969	300.0159
	-25%	0.0209	0.3421	1.8342	13663	352.0709
	+25%	0.0202	0.3451	1.8408	13686	357.4860
	+50%	0.0200	0.7399	1.6630	10257	340.4888
β	-50%	0.1436	0.2330	1.8633	14075	361.5129
	-25%	0.0200	0.7400	1.6630	10644	318.9709
	+25%	0.0201	0.0517	2.1105	16597	479.2708
	+50%	0.0200	0.3532	1.8574	13870	362.6175
δ	-50%(Approx.)	0.0203	0.1847	1.7804	13298	326.7708
	-25%	0.1313	0.1785	1.8938	14400	374.8102
	+25%	0.0200	0.1467	1.7999	13504	334.4123
	+50%(Approx.)	0.0540	0.3292	2.0330	15643	443.1035
k	-50%	0.0201	0.3474	1.8458	13796	358.4468
	-25%	0.0211	0.3501	1.8496	13810	360.1682
	+25%	0.0742	0.1555	1.9594	15047	404.8829
	+50%	0.0200	0.7401	1.6633	10592	320.0635
p	-50%	0.0200	0.7400	1.6630	11156	319.9252
	-25%	0.0201	0.3461	1.8430	13746	357.2098
	+25%	0.0204	0.3445	2.1026	16233	479.9772
	+50%	0.0200	0.7401	1.6631	9541.10	319.9854
c_2	-50%	0.0200	0.1796	1.7979	13469	333.9771
	-25%	0.0200	0.2681	1.7901	13319	332.3525
	+25%	0.0201	0.1830	1.8609	14073	360.9114
	+50%	0.0204	0.1421	2.0323	15783	440.1299
c_0	-50%	0.0300	0.1097	2.0035	15508	425.8017
	-25%	0.0202	0.1183	2.0444	15914	445.9398
	+25%	0.0200	0.6416	1.7015	11501	321.4456
	+50%	0.0200	0.0526	1.8776	14283	367.4362
I_p	-50%	0.0200	0.7399	1.6630	10624	319.9119
	-25%	0.0218	0.1457	1.9251	14718	389.1970
	+25%	0.0493	0.0587	2.1170	16663	482.6613
	+50%	0.0200	0.7399	1.6630	10615	319.9119
I_e	-50%	0.0200	0.7399	1.6630	10618	319.9119
	-25%	0.1197	0.3782	1.9737	15008	414.3762
	+25%	0.0200	0.7400	1.6630	10618	319.9252
	+50%	0.0203	0.3505	1.8518	13804	361.1871
M	-50%	0.0200	0.7399	1.6630	10618	319.9119
	-25%	0.0807	0.4084	1.8969	14183	381.0159
	+25%	0.0200	0.7399	1.6630	10618	319.9119
	+50%	0.1283	0.1389	1.8858	14340	371.2190

Table 3: Sensitivity analysis for numerous parameters of this inventory framework Case 2 $(t_1 < M \le t_2)$

Parameters	Change in parameters	t_1	t_2	T	Total Profit (TP_2)	Q
a	-50%	0.0050	0.7406	1.6635	9487.2	296.2387
	-25%	0.0050	0.8603	1.8430	11452	412.6140
	+25%	0.0050	0.7402	1.6635	12124	366.2342
	+50%	0.0053	0.3476	1.8460	16979	441.5661
b	-50%	0.0050	0.7400	1.6630	7331	218.6130
	-25%	0.0051	0.3478	1.8465	12072	315.1107
	+25%	0.0062	0.1863	2.1133	20488	594.9280
	+50%	0.0050	0.7399	1.6630	14950	460.2987
c	-50%	0.0050	0.7400	1.6630	11211	341.3385
	-25%	0.0050	0.7400	1.6630	11247	342.8611
	+25%	0.0050	0.7400	1.6630	11318	345.9689
	+50%	0.0051	0.3570	1.8650	14986	396.5558
d	-50%	0.0050	0.7400	1.6630	11388	342.6793
	-25%	0.0067	0.7378	1.6658	11365	342.7381
	+25%	0.0050	0.7400	1.6630	11155	342.6793
	+50%	0.0051	0.3529	1.8570	14749	388.0898
e	-50%	0.0053	0.3453	1.8412	14605	380.2803
	-25%	0.0058	0.3431	1.8361	14555	377.7642
	+25%	0.0059	0.3455	1.8412	14606	380.2593
	+50%	0.0051	0.1492	1.9296	15760	417.8263
α	-50%	0.0052	0.0261	2.1872	18676	558.4336
	-25%	0.0051	0.1655	1.9650	16140	435.7776
	+25%	0.0050	0.7400	1.6630	11042	354.1968
	+50%	0.0050	0.7400	1.6630	10842	365.7143
β	-50%	0.0050	0.7400	1.6630	11299	340.7980
	-25%	0.0052	0.3499	1.8510	14694	385.7024
	+25%	0.0050	0.7400	1.6630	11278	339.7147
	+50%	0.0050	0.3517	1.8547	14764	385.1202
δ	-50%	0.0051	0.0935	1.8697	15137	388.0896
	-25%(Approx.)	0.0076	0.2177	1.8582	14944	384.0574
	+25%	0.0051	0.1468	1.7999	14373	355.9653
	+50%(Approx.)	0.0051	0.0545	1.8020	14427	356.3343
k	-50%	0.0058	0.1266	1.8308	14708	369.9846
	-25%	0.0050	0.7400	1.6630	11190	342.6793
	+25%	0.0050	0.7399	1.6630	11095	342.6492
	+50%	0.0050	0.7399	1.6630	11047	342.6492
p	-50%	0.0051	0.3509	1.8531	14851	386.1307
	-25%	0.0053	0.0748	2.0202	16784	464.1106
	+25%	0.0052	0.3550	1.8611	14779	390.1171
	+50%	0.0092	0.8297	2.1660	16167	607.0093
c_2	-50%	0.0050	0.1975	1.7966	14308	355.2642
	-25%	0.0050	0.3091	1.7840	14053	352.6226
	+25%	0.0058	0.3431	1.8361	14555	377.7642
	+50%	0.0050	1.1644	1.4042	3439.7	364.8383
c_0	-50%	0.0050	1.0409	1.4975	6520.4	352.4743
	-25%	0.0052	0.1332	2.0644	17255	488.4178
	+25%	0.0051	0.0462	2.0222	16807	465.0899
	+50%	0.0050	0.5706	1.7250	12759	345.1037
I_p	-50%	0.0055	0.3481	1.8464	14658	382.8116
	-25%	0.0050	0.9051	1.8030	11692	437.1441
	+25%	0.0053	0.3453	1.8412	14605	380.2803
	+50%	0.0053	0.0295	1.9104	15588	407.6779
I_e	-50%	0.0053	0.0359	1.9899	16453	448.0494
	-25%	0.0053	0.3453	1.8412	14605	380.2803
	+25%	0.0064	0.1260	1.8404	14815	374.4475
	+50%	0.0050	0.7400	1.6630	11241	342.6793
M	-50%	0.0058	0.1266	1.8405	14815	374.5158
	-25%	0.0050	0.7400	1.6631	11242	342.7106
	+25%	0.0050	0.7400	1.6630	11241	342.6793
	+50%	0.0050	0.7400	1.6630	11242	342.6793

Table 4: Sensitivity analysis for numerous parameters of this inventory framework Case 3 ($t_2 < M < T$)

Parameters	Change in parameters	t_1	t_2	T	Total Profit (TP_3)	Q
a	-50%	0.0075	0.3507	1.8475	17576	455.4450
	-25%	0.0053	0.3439	1.8378	21165	543.3732
	+25%	0.0940	0.3503	1.8293	28426	718.2252
	+50%	0.0051	0.1197	1.9559	36159	966.0732
b	-50%	0.0951	0.1093	2.0117	22868	628.8119
	-25%	0.0050	0.7401	1.6629	16749	467.7182
	+25%	0.0063	0.3555	1.8616	26447	686.7840
	+50%	0.0050	0.7401	1.6630	21987	629.4380
c	-50%	0.0054	0.3445	1.8385	24877	636.8695
	-25%	0.1435	0.4943	1.8821	25679	678.9147
	+25%	0.0535	0.3884	1.8707	25654	668.3185
	+50%	0.0050	0.3512	1.8535	25330	654.6926
d	-50%	0.0050	0.7401	1.6630	19657	551.0151
	-25%	0.0050	0.7401	1.6630	19554	551.0151
	+25%	0.1134	0.1618	1.9555	27754	742.0520
	+50%	0.0051	0.7928	2.0183	26977	882.6639
e	-50%	0.0050	0.7862	1.6912	19670	584.9130
	-25%	0.0050	0.7400	1.6630	19449	551.0168
	+25%	0.0052	0.3438	1.8373	24922	637.9283
	+50%	0.1098	0.2060	1.9527	27674	739.4909
α	-50%	0.0050	0.3528	1.8568	25389	652.2604
	-25%	0.5251	0.7401	1.6630	20338	497.2481
	+25%	0.0050	0.7624	1.6782	19296	585.5055
	+50%	0.0050	0.7400	1.6630	18904	582.3689
β	-50%	0.0050	0.7400	1.6630	19541	547.6322
	-25%	0.0050	0.7400	1.6630	19449	551.9394
	+25%	0.0050	0.3592	1.8691	25612	666.4029
	+50%	0.0050	0.3533	1.8577	25384	654.7217
δ	-50%	0.0051	0.0933	1.8689	25819	462.1282
	-25%	0.1778	0.2562	1.9516	27622	513.5291
	+25%	0.0051	0.0534	1.8540	25493	453.0721
	+50% (Approx.)	0.0050	0.0155	1.8015	24352	422.7007
k	-50%	0.5201	0.7387	1.6629	20307	500.1346
	-25%	0.0050	0.3604	1.8713	25644	669.4748
	+25%	0.0051	0.3505	1.8521	25183	651.4613
	+50%	0.0050	0.7400	1.6630	19356	551.0168
p	-50%	0.0050	0.7401	1.6630	20401	551.0151
	-25%	0.0050	0.3580	1.8668	25614	665.3020
	+25%	0.0050	0.7400	1.6630	18907	551.0168
	+50%	0.0050	0.7401	1.6630	18410	551.0151
c_2	-50%	0.0050	0.0393	1.8125	24596	609.2351
	-25%	0.0050	0.2070	1.7958	24179	596.5540
	+25%	0.0050	0.3833	1.7715	23423	583.7035
	+50%	0.4652	0.5191	1.7400	22612	553.9315
c_0	-50%	0.0087	0.3284	1.9916	28438	783.7306
	-25%	0.0054	0.1084	2.0254	29384	813.2807
	+25%	0.0050	0.7690	1.8768	24001	737.2158
	+50%	0.0050	0.1586	1.8619	25646	653.9367
I_e	-50%	0.3549	0.4071	1.9891	28305	778.0884
	-25%	0.1133	0.3088	2.0172	29087	806.1991
	+25%	0.0050	0.7401	1.6630	19536	551.0151
	+50%	0.0050	0.7400	1.6630	19625	551.0168
M	-50%	0.0052	0.1122	2.0707	30453	861.7764
	-25%	0.0052	0.3437	1.8373	24910	637.8664
	+25%	0.0050	0.7401	1.6630	19655	551.0151
	+50%	0.0050	0.7401	1.6629	19795	550.8901

7. Observations

From Table 1, when the credit duration M is short or equal than the duration of non-deterioration i.e, ($0 < M \le t_1$) (case 1), we observe that

1. When we decrease the parameters by 25% and 50%, t_2 increases for the parameters $a, d, \alpha, \delta, p, I_p, I_e, M$ and decreases for b, β, k, c_2, c_0. T^*, TP_1 and Q^* decreases for the parameters $a, \alpha, \delta, k, p, c_0, I_p, I_e, M$, increases for d, β, c_2. For parameter b, T^*, TP_1 increases and Q^* decreases and there is no major effect on t_2, T^*, TP^* and Q^* of parameters c and e.

2. When we increase the parameters by 25% and 50%, t_2 increases for the parameters $c, d, \alpha, \beta, \delta, k, p, I_p$ and decreases for b, e, c_2, c_0, I_e, M. T^*, TP_1 and Q^* boost for the parameters $b, c, e, \delta, c_2, c_0, I_e, M$, decreases for $d, \alpha, \beta, k, p, I_p$ and for parameter a, TP_1 & Q^* increases whereas t_2 and T^* remains unchanged.

From Table 2, when the credit duration M is greater than t_1 and less or identical to t_2, the time when inventory level vanishes i. e ($t_1 < M \le t_2$) (case 2), we observe that

3. When we decrease the parameters by 25% and 50%, t_2 increases for the parameters b, d, e, β, p, c_0 and decreases for $a, \alpha, \delta, k, c_2, I_p, I_e, M$ and remains constant for c. T^*, TP_2 and Q^* increases for the parameters $e, \alpha, \delta, k, c_2, I_e, M$ whereas decreases for a, b, β, p, c_0, for parameter c, d, I_p, Q^* decreases and TP_2 increases for d, I_p, decreases for c and T^* decreases for d, increases for I_p and remains unchanged for c.

4. When we increase the parameters by 25% and 50%, t_2 increases for the parameters b, p, c_2, c_0, I_e and decreases for $a, c, d, e, \beta, \delta, I_p$ and remains constant for α, k, M. The T^*, TP_2 and Q^* rise for the parameters $a, c, d, e, \beta, \delta, p, I_p$ whereas decreases for b, c_2, c_0, I_e and there is no major effect on them of parameters k, M and for α, TP_2 decreases, Q^* increases and T^* remains unchanged.

From Table 3, when the credit period M lies between t_2 and cycle time T i.e ($t_2 < M < T$), we observe that

5. When we decrease the parameters by 25% and 50%, t_2 increases for the parameters a, e, k, p, c_0, I_e and decreases for $b, c, \alpha, \delta, c_2, M$ and remains constant for d, β. T^*, TP_3 and Q^* increases for the parameters b, e, α, c_2, M whereas decreases for $c, \delta, k, p, c_0, i_e$, for parameter d, β, TP_3 increases and T^* remains unchanged, for parameter a, TP_3 and Q^* decreases and Q^* increases whereas Q^* decreases for a and β and remains unchanged for d.

6. When we increase the parameters by 25% and 50%, t_2 increases for the parameters b, d, k, p, c_2 and decreases for $a, c, e, \alpha, \beta, \delta, c_0, I_e$ and for M remains constant. T^*, TP_3 and Q^* boost for the parameters a, e whereas decreases for $b, c, \alpha, \beta, \delta, k, c_2$, , TP_3 and Q^* decreases for p, increases for I_e and T^* remains unchanged for both p and I_e, for c_0 and M, TP^* increases, Q^* and T^* decreases vice-versa for d, TP_3 decreases whereas Q^* & T^* increases.

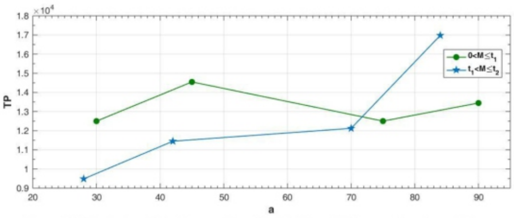

Figure 2: Effect of variation in a on the optimal total profit for case 1 and case 2 (see online version for colours)

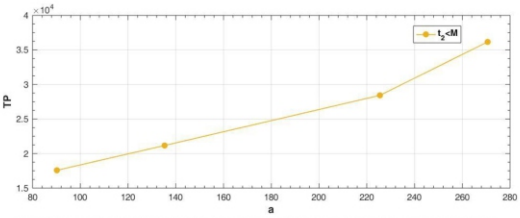

Figure 3: Effect of variation in a on the optimal total profit for case 3 (see online version for colours)

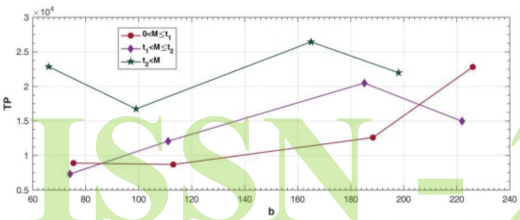

Figure 4: Effect of variation in b on the optimal total profit for all three cases (see online version for colours)

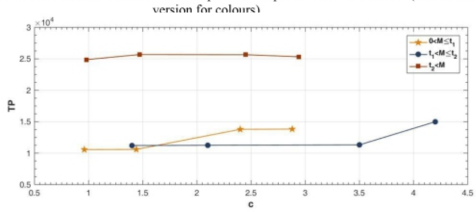

Figure 5: Effect of variation in c on the optimal total profit for all three cases (see online version for colours)

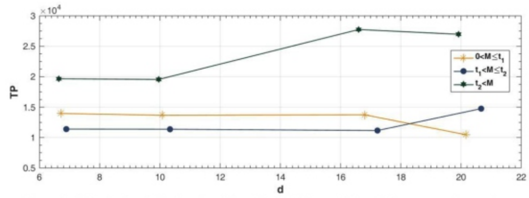

Figure 6: Effect of variation in d on the optimal total profit for all three cases (see online version for colours)

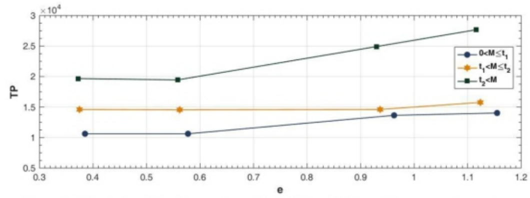

Figure 7: Effect of variation in e on the optimal total profit for all three cases (see online version for colours)

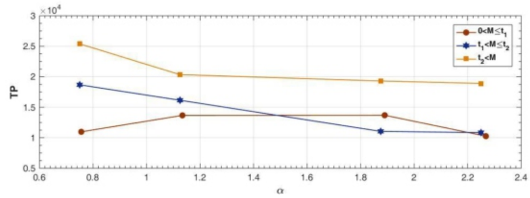

Figure 8: Effect of variation in α on the optimal total profit for all three cases (see online version for colours)

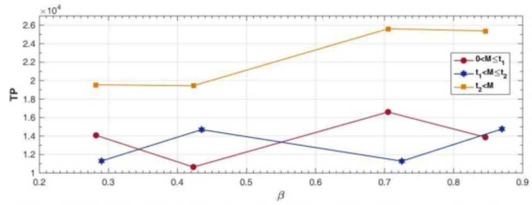

Figure 9: Effect of variation in β on the optimal total profit for all three cases (see online version for colours)

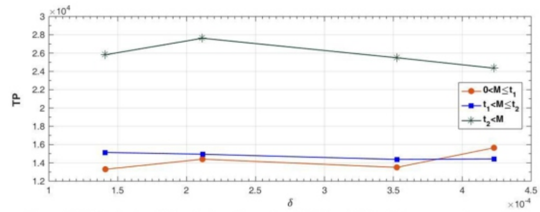

Figure 10: Effect of variation in δ on the optimal total profit for all three cases (see online version for colours)

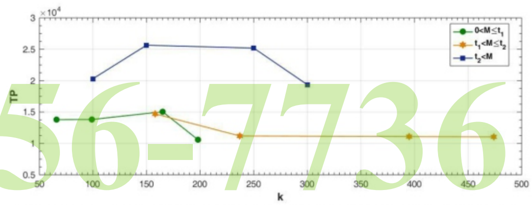

Figure 11: Effect of variation in k on the optimal total profit for all three cases (see online version for colours)

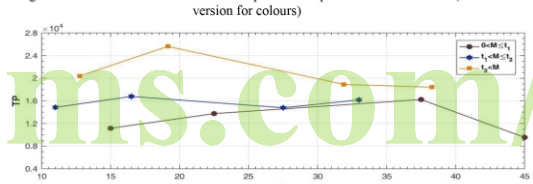

Figure 12: Effect of variation in p on the optimal total profit for all three cases (see online version for colours)

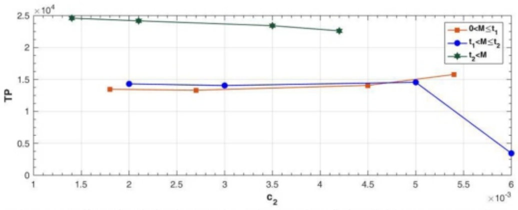

Figure 13: Effect of variation in $c_2$on the optimal total profit for all three cases (see online version for colours)

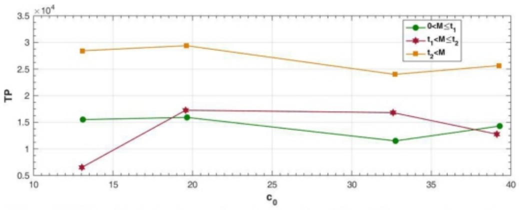

Figure 14: Effect of variation in $c_0$on the optimal total profit for all three cases (see online version for colours)

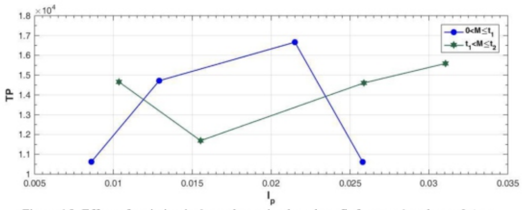

Figure 15: Effect of variation in I_p on the optimal total profit for case 1 and case 2 (see online version for colours)

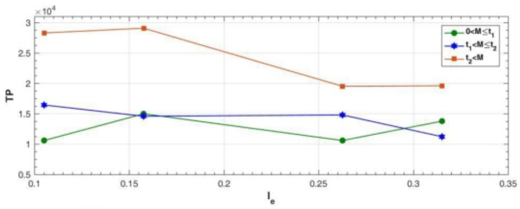

Figure 16: Effect of variation in I_e on the optimal total profit for all three cases (see online version for colours)

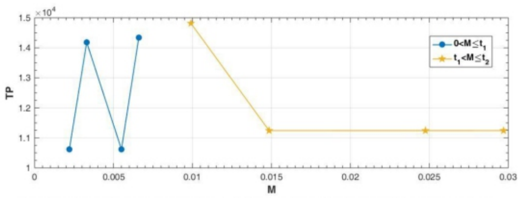

Figure 17: Effect of variation in M on the optimal total profit for case 1 and case 2(see online version for colours)

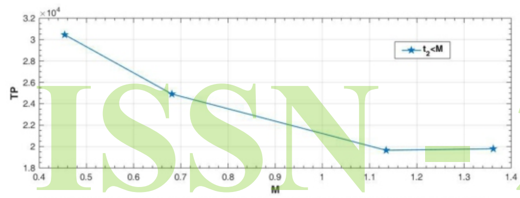

Figure 18: Effect of variation in M on the optimal total profit for case 3 (see online version for colours)

8. Conclusion

The present work offers an EOQ model for non-spontaneous deteriorating products with time dependent quadratic demand, two parameter Weibull type decline rate and time induced linear holding cost subject to trade credit policy with partial accumulation of shortages. The model is solved analytically to obtain the maximum total profit under three cases according to the hypothesis of the time after which items starts deteriorate, the time at which shortages happens, credit period and replenishment cycle length. The parameters namely $a, b, c, d, e, \alpha, \beta, \delta, p, k, c_2, c_0, I_p, I_e$ and M affects the policy making process. Numerical examples and sensitively analysis for all three cases and for all parameters is performed to minutely demonstrate the model. The main outcomes of this study are:

- The optimal values of decision variables varies sensitively with fraction of demand. By increasing the fraction of demand, we can get more profit.
- Rise in Order quantity results higher total profit.
- If we increase both the fraction of demand and order quantity at the same time, it leads maximum profit among all cases of this model.
- Longer credit period results higher total profit.

The future work in this direction can be performed by taking exponential time dependent demand or price linked demand, considering inflation.

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